Today's Outline

Lecture 9

Feb 14^{th}, 2012

I Last Lecture & Logistics

II Coordinate basis wrap up

III Light cones & World lines

IV Geodesics

(e-mail)

I LL & L

- coordinate singularities
- Local Inertial frames (Riemann Normal coords)

$$g_{\alpha\beta}(x_p) = \eta_{\alpha\beta} \qquad \left.\frac{\partial g_{\alpha\beta}}{\partial x^\gamma}\right|_{x=x_p} = 0$$

- Vectors
  - defined vectors locally
  - they live in the tangent space at P.

- different tangent spaces are completely distinct
- There are two commonly used bases:

Orthonormal basis: $\underset{\sim}{e}_{\hat\alpha}(x) \cdot \underset{\sim}{e}_{\hat\beta}(x) = \eta_{\hat\alpha\hat\beta}$

(perspective of a locally inertial observer)

Coordinate basis: $\underset{\sim}{e}_{\alpha}(x) \cdot \underset{\sim}{e}_{\beta}(x) = g_{\alpha\beta}(x)$

Tea and cookies at 3:30pm! Feb 23^{rd}, 4-5pm in 325 Le Conte

__Logistics__: Vote on HW 5:

Post	Due	Exam	Votes
Today	Feb 21	Feb 23^{rd}	0
Feb 21	Feb 28	Feb 23^{rd}	3
Today	Feb 28	Feb 23^{rd}	20

__Announce__: Eugenio Bianchi is giving a Compass Lecture titled "Black Hole Entropy and the Shape of the Horizon". Extra Credit for attending

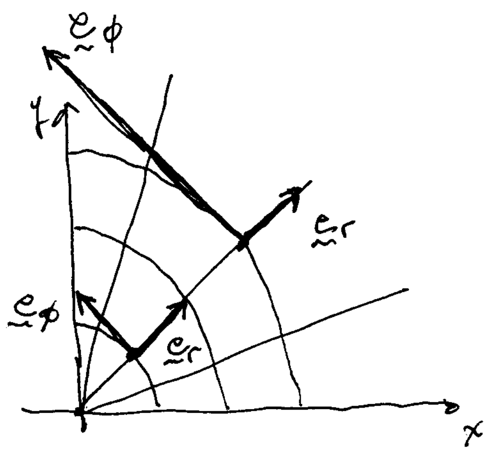
~~This is called a~~ coordinate basis.

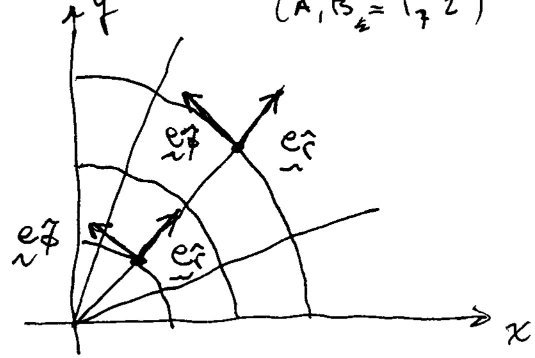
Wrap up.

Example: Plane in polar coord.s

$$ds^2 = dr^2 + r^2 d\phi^2 \iff g_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

$(A, B = 1, 2)$





Well, for a coordinate basis,

$$\begin{aligned} e_r \cdot e_r &= g_{AB}\,(e_r)^A (e_r)^B \\ &= g_{rr}\, 1\cdot 1 + g_{\phi\phi}\cdot 0 \\ &= g_{rr} = 1 \quad \Rightarrow |e_r| = 1 \end{aligned}$$

$$\begin{aligned} e_\phi \cdot e_\phi &= g_{\phi\phi} \quad \Rightarrow |e_\phi| = r \\ &= r^2 \end{aligned}$$

While for an orthonormal one,

$e_r \cdot e_r = 1$, $e_\phi \cdot e_\phi = 1$ by definition.

Important:

> Generally, coordinate bases are good for doing calculations.
>
> Generally, orthonormal bases are good for interpreting results.

What are

$(e_{\hat r})^A$ and $(e_{\hat\phi})^A$?

Well these are unit vectors, so

$$(e_{\hat r})^A (e_{\hat r})^B g_{AB} = 1$$

$$\Rightarrow (e_{\hat r})^r = 1 \qquad (e_{\hat r})^A = (1, 0)$$

$$(e_{\hat\phi})^A (e_{\hat\phi})^B g_{AB} = 1$$

$$\Rightarrow \left[(e_{\hat\phi})^\phi\right]^2 = \frac{1}{r^2} \Rightarrow (e_{\hat\phi})^A = \left(0, \frac{1}{r}\right)$$

Why would we ever consider such a strange object as $(\underset{\sim}{e}_{\hat{r}})^A$?

Well,

$$\underset{\sim}{a} = a^{\hat{\beta}} \underset{\sim}{e}_{\hat{\beta}} = a^{\alpha} \underset{\sim}{e}_{\alpha}$$

and the question "How are $a^{\hat{\beta}}$ and a^{α} related?" arises.

Answer: $\underset{\sim}{e}_{\hat{\beta}} = (\underset{\sim}{e}_{\hat{\beta}})^{\alpha} \underset{\sim}{e}_{\alpha}$

But then,

$$a^{\alpha} = a^{\hat{\beta}} (\underset{\sim}{e}_{\hat{\beta}})^{\alpha}.$$

Aside:

[Brief comments on other topics of Chap. 7:

Embedding diagrams are fun but limited

Three surfaces are important for understanding how we split spacetime into space and time

Area, 3-volume, 4-volume

$dA = \sqrt{g_{11} g_{22}}\, dx^1 dx^2$, $dV = \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3$

III Light cones & World lines

Because of the local inertial frame construction our previous definitions carry through:

$ds^2 < 0$ timelike sep.

$ds^2 = 0$ lightlike sep. (null)

$ds^2 > 0$ spacelike sep.

$$\tau_{AB} = \int_A^B \left[-g_{\alpha\beta}(x)\, dx^{\alpha} dx^{\beta} \right]^{1/2}$$

$$dv = \sqrt{-g_{00} g_{11} g_{22} g_{33}}\, dx^0 dx^1 dx^2 dx^3 \quad]$$

IV Geodesics

Variational Principle for Free Test Particle Motion

The world line of a free test particle between two timelike separated points extremizes the proper time between them.

<u>test particle</u>: This is a particle with a small enough mass that its effect on the curvature of spacetime can be neglected.

<u>free particle</u>: free of any influences except gravity

Trajectories that extremize proper time are called <u>geodesics</u>. In space these are curves that extremize the length of the curve.