Lab #7: **Rotational Motion**

Reading: Halliday/Walker/Resnick, Chapter 10 and 11.

Purpose: Study of rotations of rigid bodies. Moment of inertia, torque, and angular momentum. Conservation of angular momentum. Rolling motion. Precession.

Background: The motion of a "rigid," non-deformable object is a combination of *translation*, the motion of its center of mass, and *rotation*, the spinning of the object around an axis through the center of mass. Rotational motion is described by the angular velocity vector $\boldsymbol{\omega}$ pointing along the rotation axis, with a magnitude given by the rate of change of the rotation angle.

Each object possesses (at least) three "preferred," mutually perpendicular directions known as its *principal axes of rotation*. Along a principal axis, the direction of rotation will not change over time. (Otherwise, the motion is very complicated.) The properties of an object rotating around a principal axis are contained in a single parameter, its *moment of inertia* I.[1]

The kinetic energy of a rigid object rotating around a principal axis (with inertia I) is the sum of its translational energy and rotational energy:

$$E_{\mathrm{kin}} = \tfrac{1}{2} M v_{\mathrm{CM}}^2 + \tfrac{1}{2} I \omega^2$$

For a rolling object, the center-of-mass speed v_{CM} and the angular velocity ω are related to the radius R of the rolling object through the *rolling condition*:

$$v_{\mathrm{CM}} = \omega R$$

The direction and "amount" of rotational motion contained in a spinning object, akin to the momentum vector $\mathbf{p}$ of translational motion, is the *angular momentum* vector $\mathbf{L}$, which for an object rotating around a principal axis with moment of inertia I is given by:[2]

$$\mathbf{L} = I \boldsymbol{\omega}$$

A force $\mathbf{F}$ acting on an object can change its angular momentum, generally altering both the rotation axis and the rotation rate. If $\mathbf{F}$ is exerted at a point displaced from the center of mass by a vector $\mathbf{r}$, it will cause a *torque* $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ that changes the angular momentum $\mathbf{L}$ over time:

$$d\mathbf{L} / dt = \boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Note that the torque $\boldsymbol{\tau}$ is *perpendicular* to both the force $\mathbf{F}$ and the "lever arm" $\mathbf{r}$.

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1 In general, the three axes have different moments I_1, I_2, I_3 ("non-symmetric top"). If two axes share the same moment of inertia ("symmetric top," e. g., a solid cylinder), any axis in the same plane is also a principal axis. If all three axes have the same moment (isotropic rotator, e. g., homogeneous sphere and cube), every axis is a principal axis.

2 Otherwise, the vectors $\mathbf{L}$ and $\boldsymbol{\omega}$ are *not* parallel to each other.

In an isolated system of objects, the sum of all torques on all objects (like the sum of all forces) vanishes. Therefore, the *total angular momentum* $\mathbf{L}_{total}$, the sum over the angular momentum vectors $\mathbf{L}_k$ of the individual objects is a constant vector that does not change with time. The constancy of angular momentum, together with the conservation of energy and momentum, constitute the three fundamental conservation laws of mechanics.

In the following set of experiments, you will study different aspects of rotational motion.

Experiment #1: *Qualitative experiments with rotation.*

Unlike the more familiar translational motion, rotational motion can be quite counter-intuitive. Here is a series of little experiments, that, we hope, will surprise you a bit, and make you think about the underlying physics. No quantitative measurement is involved. (Since this is a mostly exploratory lab with many different setups, try to keep your lab report very concise.)

1. *Rolling objects*. Rolling objects of varying shape and consistency differ in the relative amount of rotational and translational energy stored in them. Unlike sliding or falling objects, rolling objects therefore *differ* in their acceleration on an inclined track. The bigger the ratio I / MR^2 (where R is the radius of the object), the slower it will move.

   In the experiment, roll pairs of objects simultaneously down the incline, and note which object "wins" the race. We'll provide you with an assortment of objects:

   - Steel balls of different diameters: ⅝", 1¼", 2½",
   - Solid balls of similar size (≈ 1¼"), but different materials: steel, aluminum, acrylic,
   - A hollow tennis ball, similar in size to the large steel ball,
   - Solid acrylic cylinders (diameter 1¼"), of lengths 1", 2", and 4",
   - A hollow acrylic cylinder (diameter 1¼"), 2" long,
   - A set of three 4" acrylic cylinders with caps, filled with air, water, and corn syrup,
   - A *Hot Wheels* toy car.

   Design your experiment to answer these questions:

   - Does the size of the object matter?
   - Does the material of the object matter?
   - Does the shape or composition of the object matter?

   Make a table with your "race" results, and sort the objects from fastest to slowest. Can you find an explanation for the behavior of the fluid-filled tubes? The *Hot Wheels* car?

   Warning: Be *very* circumspect handling the steel balls – they are heavy, and capable of causing injury and damage. (The large steel ball weighs more than 1 kg!)

2. *The gyroscope*. Gyroscopes are spinning tops that are put in a freely moving cradle to isolate them from the environment. Such a configuration can be used as a direction reference — the top will keep its rotation axis stationary in space steady even as its cradle is moved around. Can you explain why?

   Experiment with a toy gyroscope. Wind up the thread (but keep it away from the bearings!), then pull hard to spin up the gyroscope. (We'll show you how.) Hold the spinning top and move it around. Does it "willingly" follow your motion? If it has a cradle, place the top in it, and move the cradle around (carefully). Does the rotation axis of the gyroscope change? What happens if you put the spinning top on its stand?

   Now, use the gyroscope with the cradle, and try to get it out of balance on purpose. Spin up the gyroscope again, place it in the cradle, and then hang a 20 gram weight on one of the screws that hold the rotating top in place. What happens? What happens if you hang it on the opposite screw? What if you increase the weight? Over time, the speed of the spinning top goes down; how does the gyroscope react? Can you explain the observed motion (called *precession*) in terms of the torque caused by the weight, and its effect on the angular momentum vector of the top? (This is quite tricky.)

   Remove the weight and spin up the gyroscope again. Place it in the cradle, then use a hammer to give a brief vertical "kick" to the screw along the rotation axis. Can you describe the ensuing motion of the rotation axis *(nutation)? (*It may be difficult to observe with a small top.) The top quickly settles down into a new state of rotation. Describe how the rotation now differs from the rotation before the "kick."

3. *The bicycle wheel*.

   Warning: The experiment uses a large wheel. It is big & heavy, so there is a *lot* of energy stored in the wheel once it is spun up! It is capable of hurting you or destroying objects in its path if you don't handle it carefully:

   - **Always ensure that there is no one and nothing in the way when spinning it up and handling it.**
   - **Avoid contact with the wheel — it may rip your clothes and injure you.**
   - **Don't abandon it — when you're done with your experiment, brake the wheel on the floor to a standstill. Never use your hand!**
   - **If you lose control, drop the wheel and get out of the way.**

   Spin up the wheel using a sturdy string (if in doubt, ask how!): One partner holds the wheel, the other partner pulls the string as hard as possible. Then, try to move the spinning wheel around. Is this easy to do? Does the wheel actually "follow your directions"? Hold the wheel in front of you so it spins "forward" like the front wheel of a real bicycle. Try to tilt it to the left and right. What happens?

Now, hold the spinning wheel in front of you, and grab it by the attached string. Then, let the other handle go (scary, but — trust us!). What happens? Can you explain the motion in terms of torque and angular momentum?

4. *The rotational stand*. Finally, become part of the experiment and spin yourself! Step onto the low-friction rotational stand (carefully, it moves very easily). Make sure that you stand *securely* and *right in the middle* of the stand.

   First, make your partners spin you up a bit. What happens if you move your hands in and out, like an ice skater in a pirouette? Can you explain your observation in terms of your moment of inertia and angular momentum conservation? — For full effect, pick up the dumbbells, center yourself carefully, stretch out your arms, and spin up **slowly**. Then, move your arms inwards. (If you get too fast, slow down safely by stretching out your arms!) A tricky question – does your energy increase or decrease in the process?

   Now, use the bicycle wheel from the previous experiment. Let your partners spin it up and then hand it to you while you're on the stand. What happens when you try to move it around? Hold the wheel horizontally, then turn it upside down quickly (so it spins in the opposite direction). What happens? In which direction are you and the wheel now spinning? Turn it around again. Can you explain your observations?

Experiment #2: *Rotational energy and angular momentum.*

This setup uses the PASCO rotational apparatus, designed to study rotational energy and angular momentum, and measure the moment of inertia of objects. We again employ the photogate timer familiar from previous experiments.

1. *Preliminaries.* Weigh the gray cylindrical platter (no metal guides) to find its mass M. Measure its radius R. Calculate the moment of inertia for this "cylinder" $I_{cyl} = ½ M R^2$.

   Use masking tape to affix the metal strip to the other platter (with metal guides), and arrange a photogate so that the strip triggers it when the platter rotates – make sure the strip and photogate don't collide with each other or anything else as it spins around. Set the photogate timer measurement to "time" and mode to "fence", so that, once started, it will mark the time whenever the metal strip starts blocking the photogate.

   Practice determining the angular velocity ω of this platter. Spin it with your hand and start the timer. Figure out how to calculate ω from the time intervals measured.

2. *Rotational energy*. Wind up the string around the smallest of the three concentric guides (ask for assistance, there are some crucial details!), then lead it over the pulley mounted to the apparatus, and hang a 100 gram weight from the string, using the loop. Observe a "test run" of the setup: Release the platter and watch it spins up as the weight

descends; once the string is completely unrolled, the weight will fall off the apparatus.

For the actual experiment, rewind the string, and measure the height z_i of the mass above the floor. Release the platter. One partner observes the descending weight, and records the vertical position z_f at which the mass drops off. Simultaneously, the other partner activates the photogate timer, and records a couple of rotations of the platter.

Determine the vertical distance $\Delta z = z_f - z_i$ the weight traversed before falling off. Calculate the angular velocity ω of the platter. Perform the experiment three times.

For your analysis, assume that the potential energy of the weight $m\,g\,\Delta z$ is completely transformed into rotational energy $\frac{1}{2} I_p \omega^2$ of the platter. (The kinetic energy of the falling weight is almost negligible.) Determine its moment of inertia I_p.

3. *A rotational collision*. Alternatively, the moment of inertia I_p of the platter can be found using a rotational collision. Remove the string and weight, and turn the platter around so the “plain” side faces up. Rearrange the metal strip and photogate if necessary. Spin up the platter, then start the photogate timer to determine its initial angular velocity ω. After a few rotations, take the plain cylindrical platter and drop it onto the rotating platter. Record a few more rotations of the combined platters in order to find the angular velocity ω' after the collision. It is essential to measure the time intervals T and T' for a complete rotation before and after collision in a single session.

   Create a table with the rotation periods of the system. Find the periods T and T' before and after collision, then calculate the corresponding angular velocities ω and ω'.

   To analyze your data, assume that the total angular momentum L_{total} is conserved, i. e., has the same value before and after collision (why is this reasonable?):

$$L_{before} = I_p\,\omega = (I_p + I_{cyl})\,\omega' = L_{after}$$

   Determine I_p from this relation, and compare to your earlier result.

   Find the kinetic energies E and E' of the two colliding objects before and after the collision. Is this an elastic or inelastic collision?

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Reminder – Lab report: A main purpose of much of this lab was exploration, and you did a fair number of short qualitative experiments. In your lab report, try to be succinct on these experiments — briefly state what you did, what you saw, and what you learned from them. You should reserve a detailed discussion for the two more quantitative experiments.