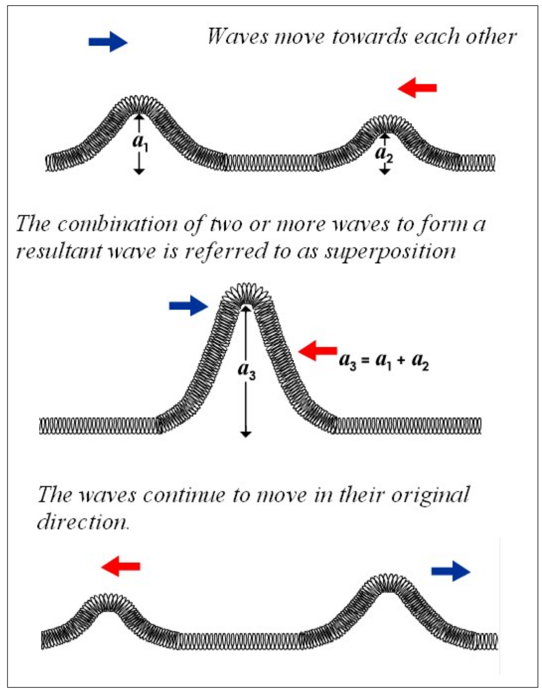
Lab #9: **Waves**

Reading: Halliday/Resnick/Walker, Chapter 16.

Purpose: Study of traveling and standing waves. Speed of a wave. Wave polarization.

Background: Earlier, we studied the oscillations of a weight on an *isolated* spring: When the weight is moved from its equilibrium position and released, it undergoes oscillations around that position. A similar disturbance in a *continuous* elastic medium does not only oscillate in time, but also spreads out in space with a characteristic speed c, the *wave velocity*.



Superposition principle. The resulting wave pattern follows from the *superposition principle*: The amplitude of the oscillation at any given point is the sum of all incoming excitations (*interference*); individual waves do not "interact" with each other.

Traveling waves. The situation is particularly simple for waves in an effectively one-dimensional medium, such as an elastic string or spring. In this case, the resulting amplitude pattern, the *wave form* $A(x, t)$, can always be interpreted as the superposition of two *traveling waves* that move with speed c to the left and to the right, respectively:

$$A(x, t) = A_+(x + ct) + A_-(x - ct)$$

Note that each traveling wave keeps its shape in space while traveling to the left or right.

Monochromatic waves. Of particular interest are the *monochromatic waves* that are generated by a disturbance (*source*) that oscillates with an angular frequency ω:

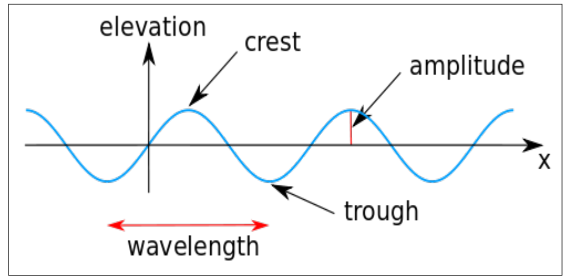
$$A(x = 0, t) = A \cos(\omega t)$$

Because the disturbance travels with speed c, the amplitude at a different point x at some time t is an image of the source at an earlier time $t - x/c$. For a wave traveling to the right,

$$A(x, t) = A(x = 0, t - x/c) = A \cos(\omega t - \omega x/c) = A \cos(kx - \omega t)$$

The inverse length $k = \omega / c$ is called the *wave number* of the wave with angular frequency ω.

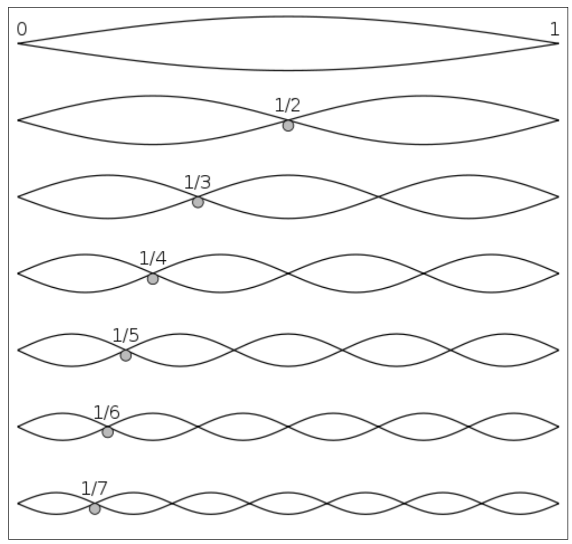
Wavelength, frequency, and period. While monochromatic waves take their most convenient mathematical form in terms of k and ω, often equivalent, more pictorial quantities are used instead – the *wavelength* λ, *frequency* f, and *period* T of the wave. They refer to the distances in space and time after which the wave repeats itself. The wave travels with speed c, so:



$$T = 1/f = 2\pi/\omega, \quad \lambda = c\,T = 2\pi\,c/\omega = 2\pi/k$$

Note that the product of wavelength and frequency yields the wave speed: $\lambda \cdot f = c$.

Standing waves. In an infinitely extended medium, waves can take on any frequency ω and corresponding wave number $k = \omega/c$. The situation is different when the length L of the medium (here, a string) is fixed. If a traveling wave encounters the "end" of the medium, it reverses direction (is "reflected"), and runs backwards. For a medium of finite length, the wave continuously bounces back and forth between the ends, and a wave can "build up" *only* if the wave passes the source always at the same "phase" in its cycle. Since the wave travels a distance $2\,L$ between each return, this distance must fit an integer multiple of the wavelength:



$$2\,L = n\,\lambda \quad (n = 1, 2, 3, \ldots)$$

In other words, a finite medium supports only a discrete set of *eigenfrequencies* f_n:

$$f_n = c/\lambda_n = n\,c/(2\,L)$$

(*quantization*). The associated wave shapes (*standing waves* or *eigenmodes*) are superpositions of waves traveling to the left and right; for a string extending from $x = 0$ to $x = L$,

$$A_n(x, t) = A \sin(n\,\pi\,x/L)\sin(\omega_n\,t)$$

Note that the wave pattern, instead of traveling, oscillates as a whole (*standing wave*), and in particular possesses $n - 1$ *nodes*, points that are always at rest, spaced half a wavelength apart from each other. Interestingly, *any* wave moving along the string can be formed by a superposition of these eigenmodes (*Fourier analysis*).

Polarization. If the disturbance is characterized by a direction, i. e., the wave amplitude is really a vector $\mathbf{A}(x, t)$, the medium will sustain different waves (often with different speeds) for different directions of the amplitude vector. Every wave then can be interpreted as the sum of two waves whose amplitudes are *perpendicular* to the direction of the wave (*transversal waves*), and one wave whose amplitude is *parallel* to the direction of the wave (*longitudinal wave*).[1]

Speed of a wave in an elastic medium. Physically, a disturbance travels along the spring because the moving piece of spring "yanks" on its neighboring parts and accelerates them. This acceleration obviously grows with the force of tension F_T in the spring, and diminishes with the weight of the spring. Hence, the wave speed c should grow with F_T, and depend inversely on the mass per length unit $\mu = m / l$ of the spring. A closer analysis shows that:

$$c^2 = F_T / \mu$$

The following experiments will (hopefully) make you more familiar with waves.

Experiment #1: *Traveling and standing waves.*

For this set of experiments, a soft metal spring ("slinky"), and a giant spring stretched across the room are provided.

First, play a bit with the slinky. Stretch it out (not too far, please) on the lab bench, then give it a little push in different directions by quickly moving one end. Can you produce a longitudinal wave? A transversal wave? Describe the patterns that you see.

The main part of the experiment deals with the long spring. For preparation, take the measuring tape and estimate its length L.

A convenient way of producing traveling wave trains in the spring is to snip it (gently!) with your finger. This yields a sudden disturbance that travels out in both directions. Generate waves and watch what happens when you snip the spring in the middle, or close to the ends.

What type of waves are they? How would you produce "horizontally polarized" and "vertically polarized" waves here, respectively?

Can you figure out what happens when a wave train is "reflected" at the ends of the spring?

Study the superposition of waves. Snip the spring twice, and watch the waves travel. Do they seem to travel at the same speed? How would you describe what happens when the reflected first wave "runs into" the second wave?

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1 This is strictly true only in *isotropic media* that lack a preferred direction in space.

Now, find the speed of the horizontally polarized wave c_H. Snip your finger at the spring near its end, and use the stopwatch to find the time t it takes the wave to travel back-and-forth for five times. Repeat this at least three times. Calculate the speed; estimate the error.

Finally, repeat the experiment for the vertically polarized wave, and find c_V. Is there any significant difference in speed between the two polarizations?

From your result, estimate the frequency $f_1 = c_H / 2L$ of the lowest eigenmode (standing wave) of the spring in horizontal polarization. Excite this eigenmode: Gently tug on the spring with a regular back-and-forth motion; start *very* slow, then slowly increase the rate. Once you hit the "right" frequency, the spring will pick up energy and oscillate with a bigger and bigger amplitude. (This phenomenon is known as *resonance*.) What is the spatial pattern of the wave? Measure its period T_1, timing five complete oscillation cycles, then find its frequency $f_1 = 1 / T_1$. Does your result agree with your prediction?

The theory stated above predicts a whole series of standing waves sustained by the spring whose frequencies are multiples of f_1. Tug again on the spring, but increase the rate of oscillation beyond the first standing wave. Can you find other resonances? Sketch the oscillation patterns you observe.

Experiment #2: *Standing waves on a string.*

Next, we examine these standing waves more quantitatively. Instead of the slinky, we will use a string that is fixed to a clamp on a table. The other end of the string is led over a pulley, and you can attach various weights at this end to control the tension T in the string.

To excite standing waves on the string, we use an *oscillator*, basically a loudspeaker membrane that drives a "stem" up and down. A *function generator* supplies an alternating electric current that controls the vertical position of the oscillator. It sends out a periodic electric signal that sets the oscillator into a harmonic up-and-down motion. Its important features are two knobs for setting the oscillation frequency f (coarse and fine control), a knob to adjust the "output level," i. e., the amplitude of the oscillation, a series of buttons to control the frequency range, and a digital display that shows the current oscillation frequency.

Finding the eigenfrequencies of a string. Attach a 1 kg mass to the free end of the string. Make sure the oscillator is in the "unlocked" position, and that the string is attached to the moving oscillator "stem." Measure the string length L between the oscillator stem and the pulley.

Select "sine wave" for the "wave form," then switch on the function generator. Initially, pick a low frequency range (1 Hz) to understand the operation of the setup: Watch the motion of the stem while you play around with the controls of the generator.

To start the first experiment, select a low-to-medium output level, and *slowly* increase the oscillation frequency until the string "picks up" the motion of the oscillator and starts to vibrate intensely as a whole, in a way similar to the giant slinky. (You may have to switch the frequency range to 10 Hz.) The resonance frequency f_1 is sharply defined – adjust the frequency carefully to find the maximum response of the string. Write it down.

Then, increase the frequency further, until you "hit" the next resonance. The corresponding standing wave with $n = 2$ should feature two maxima of the amplitude (*antinodes*), separated by a motionless *node* in the center of the string. Again, home in to the resonance frequency f_2, and record it. Keep increasing the oscillation frequency until you have found the first ten resonances of the string. (You may have to adjust the output level of the generator.) Create a table displaying the index n of the standing wave, and its eigenfrequency f_n.

Theory predicts that the resonance frequencies f_n grow linearly with the index n:

$$f_n = n\, c \,/\, (2\, L)$$

To test this hypothesis, plot f_n as a function of n. Do the data points fall on a straight line? Find a best line fit, and extract its slope. Use it to calculate the speed c of the wave.

The relation between tension and wave speed. In the second part of this experiment, we will use standing waves to examine how the speed of the wave c depends on the tension F_T in the string. In the setup, the tension is adjusted by changing the mass m hanging from the free end of the string, and we will find the corresponding resonance frequency f_n for a fixed index n of the standing wave – a value around $n = 5$ is a reasonable choice. Record your value for n.

To start the experiment, remove the 1 kg weight from the string, and replace it with a 100 gram mass instead. Start the oscillator and adjust the frequency, until the string displays your chosen standing wave. Note down the resonance frequency, then replace the 100 gram mass by a 200 gram mass, and repeat the procedure. Add weight in 100 gram steps until you have reached again 1 kg as attached mass.

Create a table that contains:

- the attached mass m,
- the corresponding tension in the string (weight of the mass) $F_T = mg$,
- the resonance frequencies f_n,
- the corresponding wave speed $c = 2\, L\, f_n \,/\, n$ (see above), and
- the square of the wave speed c^2.

Theory suggests that the square of the wave speed c^2 grows linearly with the tension F_T:

$$c^2 = F_T \,/\, \mu$$

Test this hypothesis: Plot c^2 as a function of the tension F_T in the string. Do the data points follow a straight line? Again, add a best-fit line, and extract its slope. Its inverse should be the *mass density* μ (mass per length) of the string.

To check this prediction, measure the mass per length of string μ directly: Weigh a prepared one meter piece of string, using the electronic precision scale. Do the two values match? If not, what could be the reasons for the discrepancy?

Experiment #3: *Observing standing waves with a stroboscope.*

In the previous setup, the oscillation of the string is too fast to follow for your eye – all you can see is a blurred image of the vibrating string. To observe the shape of the oscillating string, we use a *stroboscope*, a device that emits brief flashes of light at a regular, adjustable interval. If the repetition rate of the stroboscope matches the oscillation frequency of the string, it appears to stand "still" (can you explain, why?), and the wavelike pattern becomes visible.

A simple assembly, using a vibrator similar to an old-style door bell, is placed at a dark location. The electromagnet in the apparatus attracts a steel blade in the rhythm of the alternating (AC) current provided by the electric grid, and generate waves in the attached string. Its tension has been chosen so the string is in resonance.

Observe the standing wave using the stroboscope, and verify that the motion of the string seems to cease near a repetition rate of 120 flashes per second (twice the frequency of the grid). What happens when you slowly vary the repetition rate? Predict the image generated at various repetition rates (240 Hz, 360 Hz, 60 Hz, 40 Hz), then test your prediction.

Question. All of this has a lot to do with music. What happens when you "tune" a musical string instrument, like a guitar or violin? Which strings on a harp yield the higher notes? Why is a double bass bigger than an ukulele? How do the six strings of a guitar differ?

REMINDER: CAFE/Evaluation forms!