Homework 1

Due Wednesday, September 11, 2019

Reading. This week: Chapters 1 & 2, and Carey's *The Hidden Value of Ignorance*. Listen to Radiolab ≤kg. Next week: Ch. 2.

Homework assignments are designed to prepare you for doing physics. All homework assignments are due in a week. Each homework consists of three parts:

1. **Theory [10 points]**: This part gives you an opportunity to review a central concept or formula discussed in class. You are expected to write a detailed derivation from first principles. The derivation is guided through a number of steps where you are asked to provide detailed explanations of assumptions, methods, results and dimensional-analysis checks. The grade evaluates the completeness of the work shown and the conceptual clarity of the application of the principles.

2. **Exercises [10 points each]**: This part consists of textbook-style exercises. The topic of the exercises is directly related to the topic covered in the Theory part. You are expected to write a derivation of the analytical solution and its numerical value (reported in a box). The grade of each exercise evaluates the correctness of the analytical and numerical solution of each exercise.

3. **Problem [10 points]**: This part consists of a concrete physics problem that requires identifying various layers of simplifying assumptions and order of magnitude estimates. We will vary between descriptions of everyday phenomena and analysis of historically important experiments. Having solved the part 1. (Theory) gives you some of the necessary tools to attack the problem. The problem is stated in everyday language and no values are given to you. You are expected to make a reasonable estimate of a requested quantity using all information and methods you have in your toolbox from current and previews homework assignments in the course. The grade evaluates the completeness of the work shown, the conceptual clarity in the descriptions of the assumptions and methods used, correctness of the order of magnitude estimated and relative checks of consistency.

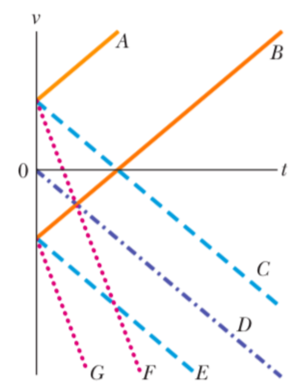
1. **Theory**: The fundamental theorem of calculus states the following relation between the derivative and the integral of a function:

$$\frac{dF(t)}{dt} = f(t) \qquad \Longleftrightarrow \qquad F(t) = F(t_0) + \int_{t_0}^{t} f(\tau)d\tau. \tag{1}$$

(a) Check the validity of this theorem for a polynomial function of t. [Hint][1]
Use the definitions $v(t) \equiv \frac{dx(t)}{dt}$ and $a(t) \equiv \frac{d^2x(t)}{dt^2}$ to answer the following questions:
(b) A particle has velocity $v(t) = v_0 + a_0 t$. Compute the position $x(t)$ as a function of time.
(c) A particle has acceleration $a(t) = -g$. Compute its velocity $v(t)$ as a function of time.
(d) A particle has acceleration $a(t) = -g$. Compute its position $x(t)$ as a function of time.

Exercises:

2. Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in the figure at right give the velocity $v(t)$ for (a) the dropped egg and (b) the thrown egg? (Curves A and B are parallel; so are C, D, and E; so are F and G.)



Comparisons for Prob. 1.

3. The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s, at which he commented, "Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Hubers record by 19.0 km/h. What was Whittingham's time through the 200 m?

4. Two trains, each having a speed of 25 km/h, are headed at each other on the same straight track. A bird that can fly 50 km/h flies off the front of one train when they are 50 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

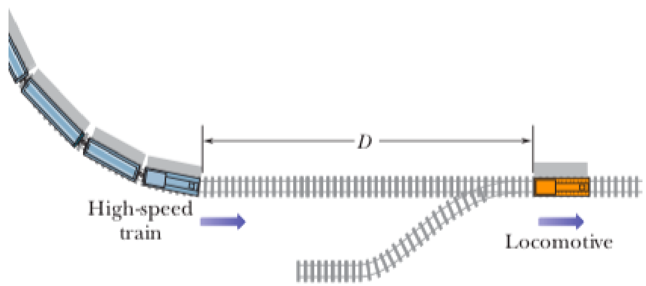
5. The position function $x(t)$ of a particle moving along an x-axis is $x = x_0 - a_0 t^2$, with x in meters and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin $x = 0$? (e) Let $x_0 = 4$ m and $a_0 = 6$ m/s^2, graph x versus t for the range -5 s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops?

6. The position of a particle moving along the x-axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t in seconds. What are the units of (a) constant c and (b) constant b? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the

[1] In order to check the $\Longrightarrow$ implication, we assume that $F(t) = at^n$ and define $f(t)$ as its derivative, i.e. $dF(t)/dt = f(t)$. Then we check that, with these assumptions, the right hand side of Eq. (1) is satisfied. To check the $\Longleftarrow$ implication, we start from the right hand side and define $f(t) = bt^k$. Finally, argue that any polynomial is a sum of terms like these, and hence the theorem holds for any polynomial because integrals and derivatives distribute over sums.

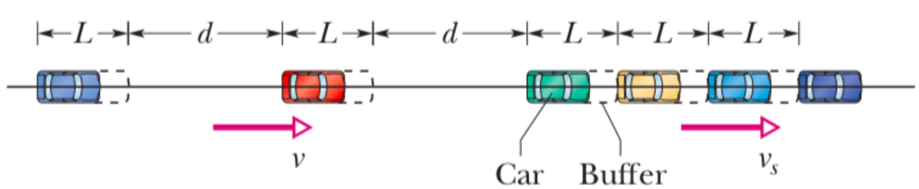
particle reach its maximum positive x position? From $t = 0.0$ s to $t = 4.0$ s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, and (g) 2.0 s. Find its acceleration at times (h) 1.0 s, and (i) 3.0 s.

7. When a high-speed passenger train traveling at $v_T = 163$ km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 600$ m ahead, (see Figure below). The locomotive is moving at $v_L = 29.0$ km/h. The engineer of the high-speed train immediately applies the brakes. (a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch $x(t)$ curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.



Setup for Exercise 6.

8. **Problem**: *Traffic shock wave.* An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Checkout this video to see this striking phenomenon. (a) Which direction is the shock wave traveling (downstream or upstream) for the highway on the right of the video? The figure below models this situation. It shows a uniformly spaced line of cars moving at speed v toward a uniformly spaced line of slow cars moving at speed v_s. Assume that each faster car adds length L (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (b) Provide reasonable estimates from your knowledge of cars and highways for L, d, v, and v_s. (c) With your estimates from (b), what is the speed and direction of travel of the shock wave? Is your answer consistent with the video above? (d) For what separation distance d between the faster cars does the shock wave remain stationary? Give a symbolic answer before you plug in your numbers. (e) If the separation is twice what you found in part (d), what are the speed and direction (upstream or downstream) of the shock wave?



Setup for Problem 8.