Intro Physics
Day 11

<u>Today</u> 0. Remind Exam, remind OHs, HW

I Last time

II A Theory of Interactions: Momentum

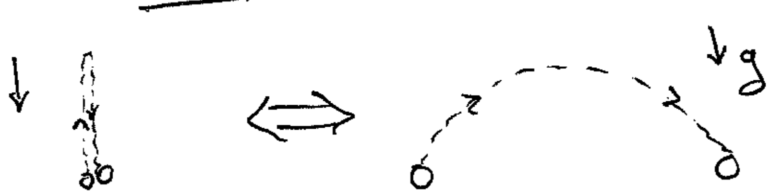
I • First a familiar fact: We can choose any time to be $t=0$ and any pt to be $x=0$, $y=0$ (and $z=0$); the rules of motion are the same, e.g.

$$y(t) = y_0 + v_{0y}t + \tfrac{1}{2}at^2$$

for constant acceleration.

or

• More Surprisingly: We can also choose any constant speed (inertial) frame to be $v=0$! The motion in different frames looks different, but the <u>rules</u> are the same



This actually implies Galileo's principle of inertia:
"Non-interacting objects maintain constant velocity"
This is true in all inertial frames
(though the constant $\vec{v}$'s may differ).

- We found how to transform between frames

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}.$$

Here

A is frame A e.g. Ball's frame, boat's frame, bird's frame

B is frame B e.g. skateboarder's frame, river frame, wind frame

C is frame C e.g. ground frame

$\vec{v}_{AC}$ is the velocity of frame A w.r.t. frame C and similarly for $\vec{v}_{BC}$ and $\vec{v}_{AB}$.

For example, the velocity of the ball w.r.t. the ground.

Note that a choice of coordinates is essential, e.g. for sign $\vec{v}_{AC}$.

- Principle of Galilean Relativity:

"The rules of motion are the same in all inertial frames."

This is a goal, but a nice one if possible. We'll have to see.

Most importantly, if we take this as a principle we can start to develop a theory of interactions.

II Consider a simple interaction: two identical objects moving toward each other at the same speed collide head on

$$\circ \xrightarrow{v_0} \qquad \xleftarrow{v_0} \circ$$

What happens?

It turns out that there are three possibilities:

"Elastic collision"

$t<0$: $\circ \xrightarrow{v_0}$ $\xleftarrow{v_0} \circ$

$t=0$: collision

$t>0$: $\xleftarrow{v_0} \circ$ $\circ \xrightarrow{v_0}$

reverses velocities

Idealized, but close to accurate for hard, elastic objects (billiard or bouncy balls)

"Realistic inelastic collision"

$\circ \xrightarrow{v_0}$ $\xleftarrow{v_0} \circ$

collision

$\xleftarrow{v_1} \circ$ $\circ \xrightarrow{v_1}$

$v_1 < v_0$

Most realistic collisions

"completely inelastic collision"

$\circ \xrightarrow{v_0}$ $\xleftarrow{v_0} \circ$

collision

$\circ\circ$

$v=0$

$\longrightarrow x$

Let's reconsider these collisions from a frame moving $\xleftarrow{} v_0$ (add $+v_0$ to all vectors)

Elastic

$t<0$: $\circ \xrightarrow{2v_0}$ $\overset{v=0}{\circ}$

$t>0$: $\overset{v=0}{\circ}$ $\circ \xrightarrow{2v_0}$

collision transfers, but preserves velocity.

Is velocity always preserved?

Inelastic

$\circ \xrightarrow{2v_0}$ $\overset{v=0}{\circ}$

$\circ \xrightarrow{v_0 - v_1}$ $\circ \xrightarrow{v_0 + v_1}$

Before: $2v_0$

After: $v_0 - v_1 + (v_0 + v_1)$
$= 2v_0$

Completely Inelastic

$\circ \xrightarrow{2v_0}$ $\overset{v=0}{\circ}$

$\circ\circ \xrightarrow{v_0}$

Velocity is certainly not preserved!

We see from the table above that velocity is not always preserved! But, the idea of looking for something that is preserved is profound. The change in speed in the last example seems to be related to the doubling of the amount of stuff. This is correct and we characterize it quantitatively by the <u>mass</u> of the object. We introduce <u>momentum</u>

Momentum: $\vec{p} = m \cdot \vec{v}$

mass times velocity

Inelastic:

$$\vec{P}_{tot\,i} = 2\vec{v}_0 m + 0 = 2m\vec{v}_0$$

and

$$\vec{P}_{tot\,f} = m(\vec{v}_0 - \vec{v}_1) + m(\vec{v}_0 + \vec{v}_1) = 2m\vec{v}_0$$

Yes!

Completely inelastic:

$$\vec{P}_{tot\,i} = m(2\vec{v}_0) + 0$$

and

$$\vec{P}_{tot\,f} = 2m(\vec{v}_0) = 2m\vec{v}_0$$

Yes, again!

Let's check if momentum is preserved: first elastic

$$\vec{P}_{tot\,i} = m(2\vec{v}_0) + m(\underset{\text{zero speed}}{0}) = 2m\vec{v}_0$$

initial

and

$$\vec{P}_{tot\,f} = m(0) + m(2\vec{v}_0) = 2m\vec{v}_0$$

Yes(!) it's preserved. Let's check the other two cases as well:

This leads to our first theory of interactions: they preserve the total momentum in the system! We call this "conservation of momentum" and it is a <u>profound result.</u>

We'll build up the rest of our results from this.