Homework 1

Due Tuesday, September 7th, 2021

Reading. This week: Chapters 1 & 2 from Halliday, Resnick and Walker's *Fundamentals of Physics*, and Carey's *The Hidden Value of Ignorance*. Listen to Radiolab ≤kg. Next week: Ch. 2.

Homework assignments are designed to prepare you for doing physics. All homework assignments are due in a week. Each homework consists of three parts:

1. **Theory**: This part gives you an opportunity to review a central concept or formula discussed in class. You are expected to write a detailed derivation from first principles. The derivation is guided through a number of steps where you are asked to provide detailed explanations of assumptions, methods, results and dimensional-analysis checks. I am looking for the completeness of the work shown and the conceptual clarity of the application of the principles.

2. **Exercises**: This part consists of textbook-style exercises. The topic of the exercises is directly related to the topic covered in the Theory part. You are expected to write a derivation of the analytical solution and its numerical value (reported in a box). I am looking for the correctness of the analytical and numerical solution of each exercise.

3. **A problem**: This part consists of a concrete physics problem that requires identifying various layers of simplifying assumptions and order of magnitude estimates. We will vary between descriptions of everyday phenomena and analysis of historically important experiments. Having solved the part 1. (Theory) gives you some of the necessary tools to attack the problem. The problem is stated in everyday language and no values are given to you. You are expected to make a reasonable estimate of a requested quantity using all information and methods you have in your toolbox from current and previews homework assignments in the course. I am looking for the completeness of the work shown, the conceptual clarity in the descriptions of the assumptions and methods used, correctness of the order of magnitude estimated, and relative checks of consistency.

1. **Theory**: There are five invaluable techniques that you can use on almost every problem that you do in this course. These are: Units, Limits, Order of Magnitude, Symmetry, and Mnemonics. I will constantly ask you to apply or construct one of these methods throughout the course. It should nearly become a habit to apply every one of them to each of your problems. (I'm exaggerating a little, but not a lot.) The exercises below introduce and give you practice with these methods.

Units: The units of physical quantities can often be used to identify whether an equation or an answer that you've gotten even has a chance of making sense. Suppose you and some friends are trying to find a physical distance d in terms of a measured time t and two measured speeds v_1 and v_2. Unfortunately, you've come up with six different answers to the problem, which are listed below as (i)-(vi).

(**a**) Use units to identify which of these answers cannot possibly be correct:

$$(i)\ d = v_1 t + \frac{v_1 + v_2}{2} t\ , \qquad (ii)\ d = v_1 \left(\frac{v_1}{v_2^3}\right) t\ , \qquad (iii)\ d = \sqrt{v_1^2 + v_2^2}/t$$

$$(iv)\ d = v_1 e^{-v_2/v_1} t\ , \qquad (v)\ d = v_2 t \cos(v_1)\ , \qquad (vi)\ d = (v_1 + v_2) t \left[\ln(v_2) - \ln(v_1)\right].$$

Explain why the incorrect expressions do not make sense.

Limits: When you have an equation that describes a physical process, it is often quite valuable to explore what happens when the variables of the equation take on special values; some examples might be 1, 0, ∞ (which really means the limit of a large value for that variable), -1, initial values, etc.

(**b**) Consider the constant acceleration equation, $v^2 = v_0^2 + 2a(x - x_0)$, that we will derive in class this week. Suppose the left hand side v was equal to v_0, what two possible physical situations would give rise to this value? (Use both equations and words to explain your answer to this question.) Are these physical situations equivalent or different? Choose two other variables in this equation and explore the consequences of each of these variables taking on two special values. Be sure to explain your results with both equations and words.

Take note! In order to apply either of these methods of checking units or taking limits, your equations need to be expressed in terms of symbols. Otherwise how would you know what the units of that quantity are or what would happen to the equation when you took a limit? For this reason it is an extremely good practice to keep all of your problem solving in terms of symbols until you think you have a final answer and only then to plug in any numbers that were given in the problem.

Order of Magnitude: Sometimes physicists will behave as if order of magnitude estimates are obvious. I think this is deceptive; while the combination of the numbers involved in these estimates is often simple, you have to have a sense of lots of different physical scales to get started. We'll work on building this background knowledge throughout the semester.

(**c**) Two types of physical quantities that we'll be working with a lot are distances and times. Picking objects or physical processes that are interesting to you, draw a distance line with 5 distances each separated by at least one order of magnitude and a time line with 5 time scales each separated by at least one order of magnitude. Hold on to your lines so that you can keep adding to them over the course of the semester.

Symmetries: Many physical problems have symmetries. The symmetries can take many different forms and it is always a good idea to keep an eye out for them. Below I ask you to analyze one such example in terms of your intuition and mathematically.

(**d**) Suppose that you throw a ball straight up with some initial velocity v_0. It travels up to some maximum height, where it momentarily comes to rest, and then falls back down to your hand. Intuitively, does the upward or the downward trip take longer or do they take the same amount of time? How do you know? Again, intuitively, is the speed of the ball greater right when it leaves your hand or right before it returns to your hand, or are they the same speed? How do you know?

This is a constant acceleration problem. Use the equation $v^2 = v_0^2 + 2a(x - x_0)$ that we will derive in class to argue that the speed just after the ball leaves your hand and just before it returns to your hand are indeed equal. Use the equation $v = v_0 + at$ to argue that the upwards and downwards travel times are equal. Your expectations from your intuitive explanations above make it much easier to use the equations to prove these conclusions. This is one of the many advantages of noticing a symmetry.

Mnemonics: These are simple tools that help you to remember things. The trick where I remembered (almost) everyone's name on the first day of class was based on a mnemonic. The phrase "flip and multiply" is a mnemonic designed to help you remember how to divide two fractions:

$$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}.$$

It instructs you to take the denominator, flip it upside down, and multiply it by the numerator to get the result. A very common mnemonic strategy is to convert what you want to remember into a silly or vivid image. For example, if you want to memorize a grocery list consisting of carrots, eggs, and marshmallows, you might imagine the Stay-Puft marshmallow giant from *Ghostbusters* dropping eggs onto your newly quaffed head of carrot hairs. It's ridiculous, but I bet that I can ask any of you for that grocery list next week and you'll be able to remember it.

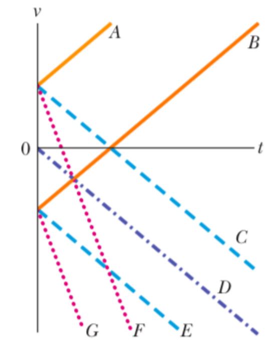
(**e**) There are certain things that we will use so often in this course that you will save yourself a lot of time by memorizing them. One example is the derivative of a power, that is,

$$\tfrac{d}{dx}\left(x^n\right) = nx^{n-1}.$$

If you don't already have this formula memorized, come up with a silly image that helps you to memorize it. To do this, you will probably first want to turn the symbols into images and then convert those images into some kind of action that helps you to remember the equation. If you already have this equation memorized, find another equation that will be useful from our textbook and use this method to memorize it instead. In either case, write out a description of your equation and image.
Take note that this technique is most useful for getting the equation into your short term memory. Over time, if you use the equation regularly, you can skip calling up the image in your mind and go straight to the equation. The image just helps you get it into your head in the first place.

Exercises:

2. Hanging over the railing of a bridge, you drop an egg (no initial velocity) as you throw a second egg downward. Which curves in the figure at right give the velocity $v(t)$ for (a) the dropped egg and (b) the thrown egg? (Curves A and B are parallel; so are C, D, and E; so are F and G.)



Comparisons for Exer. 2.

3. The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s, at which he commented, "Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber's record with an average speed that was 19.0 km/h faster than Huber's. What was Whittingham's time through the 200 m?

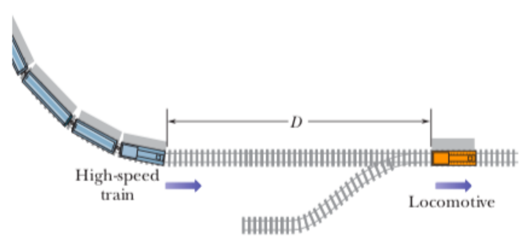
4. Two trains, each having a speed of 25 km/h, are headed at each other on the same straight track. A bird that can fly 50 km/h flies off the front of one train when they are 50 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?

5. The position function $x(t)$ of a particle moving along an x-axis is $x = x_0 - a_0 t^2$, with x in meters and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin $x = 0$? (e) Let $x_0 = 4$ m and $a_0 = 6$ m/s^2, graph x versus t for the range -5 s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops?

6. The position of a particle moving along the x-axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t in seconds. What are the units of (a) constant c and (b) constant b? Let their numerical values be 3.0 and 2.0, respectively. (c) At what time does the particle reach its maximum positive x position? From $t = 0.0$ s to $t = 4.0$ s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1.0 s, and (g) 2.0 s. Find its acceleration at times (h) 1.0 s, and (i) 3.0 s.

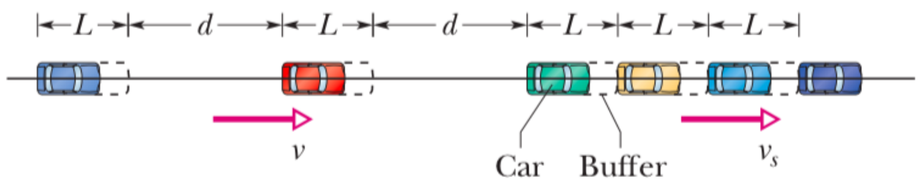
7. When a high-speed passenger train traveling at $v_T = 163$ km/h rounds a bend, the engineer is shocked to see that a locomotive has improperly entered onto the track from a siding and is a distance $D = 600$ m ahead, (see Figure below). The locomotive is moving at $v_L = 29.0$ km/h. The engineer of the high-speed train immediately applies the brakes.

(a) What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided? (b) Assume that the engineer is at $x = 0$ when, at $t = 0$, he first spots the locomotive. Sketch $x(t)$ curves for the locomotive and high-speed train for the cases in which a collision is just avoided and is not quite avoided.



Setup for Exercise 7.

8. **Problem**: *Traffic shock wave.* An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Checkout this video to see this striking phenomenon. (a) Which direction is the shock wave traveling (downstream or upstream) for the highway on the right of the video? The figure below models this situation. It shows a uniformly spaced line of cars moving at speed v toward a uniformly spaced line of slow cars moving at speed v_s. Assume that each faster car adds length L (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (b) Provide reasonable estimates from your knowledge of cars and highways for L, d, v, and v_s. (c) With your estimates from (b), what is the speed and direction of travel of the shock wave? Is your answer consistent with the video above? (d) For what separation distance d between the faster cars does the shock wave remain stationary? Give a symbolic answer before you plug in your numbers. (e) If the separation is twice what you found in part (d), what are the speed and direction (upstream or downstream) of the shock wave?



Setup for Problem 8.