Spin Chains and Lattice Field Theory with Python Tutorial

By: Cole Cecchetto
with some input from Hal Haggard

This is a provisional draft for the schedule and proposal for the content of the planned Python tutorial in the spring semester 2020.

Proposed Content:

This tutorial is planned such that it can be separated into two halves distinguished from one another by their content, with the first focusing on spin chains and the second focusing on field theories on the lattice, with both focusing on computations in Python.

Overall Schedule & Time Constraints:

We plan to meet roughly 15 times during the semester, and may do a few extra meetings provided there is opportunity. Below is a preliminary sketch of an explicit schedule.

Lattice Field Theories:

Lattice Field Theory: An Undergraduate Introduction by Leon Hostetler is a short, 30-page set of notes on a first time encounter with field theory on the lattice. It has great notes on the physical ideas of the Ising model and a ready-made Monte-Carlo algorithm in Python. Another text for this is *Monte Carlo Applications and Lattice QCD* by Jacob L. Scott; despite the title, this short work touches on every physical topic of this tutorial.

After covering the basic sections of this topic, there will be an investigation of *Hamiltonian formulation of Wilson's lattice gauge theories* by Kogut and Susskind. This was a very important paper on the topic of field theory on the lattice and it is cited to this day. As such it is worth spending about 3 weeks pouring over the ideas in this paper. There should be some attempts to try and implement the ideas of this paper, but it was written in the 1970s and there is no code, certainly no python code, in the paper itself. As a consequence, it may be worthwhile to read more recent works against the text if implementation becomes the overwhelming goal.

Quantum Many-Body Systems:

There will be a short section in this tutorial on the spin chain, its spectrum, and quantum chaos. This has been constructed around two core texts, *An introduction to the spectrum, symmetries, and dynamics of spin-1/2 Heisenberg chains* by Kira Joel, Davida Kollmar, and Lea F. Santos and *Quantum chaos: An introduction via chains of interacting spins 1/2* by Aviva Gubin and Lea F. Santos; both of these articles were published in the AJP. The goal of this section is to translate the given Mathematica code into python code and explore how quantum chaos emerges and explore the spin chain's spectrum. The plan is for this to take about 2 weeks.

Week 1: Statistics & basic python competency

The point of this class is to get situated & review the concepts of probability and statistics as presented in 3.1 of *THE MONTE CARLO METHOD IN QUANTUM FIELD THEORY* by COLIN MORNINGSTAR. In addition to this, the first class would be an opportune moment to make sure operations like integration and differentiation are understood in Python.

Week 2: Monte Carlo Integration

The text is section 3.2 of *The Monte Carlo Method In Quantum Field Theory* by Colin Morningstar. The goal should be to perform a few Monte Carlo integrations in python.

Week 3: Continuing Monte Carlo; stochastic processes and Markov Chains

The text is sections 3.2 & 3.3 of *The Monte Carlo Method In Quantum Field Theory* by Colin Morningstar & Anders Tolver's *An Introduction to Markov Chains*; in addition to this, a very helpful document with some python code is the 12th lecture of a scientific computing class whose hyperlink is the following: https://rein.utsc.utoronto.ca/teaching/PSCB57_notes_lecture11.pdf Compared to the previous week, there is about twice the amount of text to process, so the amount of actual coding should probably less. A tenable goal for this week's homework would be constructing a transition matrix, exploring the correlation function given in the code, and perhaps writing out a diagram for what a Markov Chain really is.

Week 4: Introduction to the Ising Model of Ferromagnetism

The basic texts for this week will be the first half of *Lattice Field Theory: An Undergraduate Introduction* by Leon Hostetler and section 8.2 of Daniel Schroeder's *Thermal Physics*, *The Ising Model of a Ferromagnet*. The goal of this week will be to understand how the Ising model works, how the interaction between spins works, et cetera. If explicit homework is desired for this week, then the following problems from Schroeder should do. 8.15, 8.19—8.21.

Week 5: Ising; Setting up a lattice, measuring magnetization, & looking at the phase transition

The basic texts for this week are those of the previous week. The Hostetler has a program for python that assembles a lattice and allows for ready-made simulations. The goal of this week would be to understanding how this works, inside and out.

Week 6: Ising; constructing a Monte Carlo algorithm from the ground up

Using the aforementioned texts the objective for this week is to reconstruct a Monte Carlo algorithm from the ground up; if this is not sufficient as a task, then perhaps the construction could be in a different dimension.

Week 7: Ising; Correlation functions and correlation lengths

The texts for this week are the Hostetler and Schroeder from the previous weeks and what else would be a useful resource for understanding correlation functions and their relation to correlation lengths. The explicit homework for this week could be Schroeder's 8.29.

Week 8: The physical and mathematical ideas behind the path integral

The point of this week is to establish an understanding of what the path integral is in the context of quantum mechanics, rather than quantum field theory. The second, third, and fifth chapters of *Path Integral Methods and Application* by Richard Mackenzie introduce the path integral in its various formulations, derive its connection to the principle of least action, and the the imaginary time/partition function flip side, respectively.

Week 9: Discretizing the path integral & an application to the quantum harmonic oscillator in 1D

With the ideas of path integrals under command, this week will focus on the application of them. The requisite reading will be on discretizing the path integral, programming this into python, and applying this to the quantum harmonic oscillator. The reading for all this will be chapters 2 and 4, with reference to 3 if necessary, of Jacob L. Scott's *Monte Carlo Applications and Lattice QCD*. The homework will be reproducing figure 5 page 13, working out how Monte Carlo simulations can generate measurements of the lattice and how the Metropolis algorithm can get at energy excitations. An additional reference that may be useful if needed would be *Lattice QCD for Novices* by G. Peter LePage.

Week 10: Free Week

This week is free on account of days off.

Week 11: Introduction to Quantum Many-Body systems

The reading for this week is two articles: Martin Gutzwiller's *Quantum Chaos* and Lea Santos' *Quantum chaos: an introduction via spin chains of interacting spins 1/2*. The first of these is a general introduction to the idea of quantum chaos that is largely non-technical. The second is a technical paper that has an accompanying Mathematica code. The homework for this week will be to try and reverse engineer the aforementioned Mathematica code in python.

Week 12: Constructing the spectrum and simulating quantum chaos

The reading for this week is *An Introduction to the spectrum, symmetries, and dynamics of spin-1/2 Heisenberg chains* by Lea Santos. The tasks for this week should be to explore two of the three topics mentioned: dynamics could be explored via the the time evolution of probabilities, the spectrum can be explored via the energy bands and how the states are distributed, and the symmetries can be gleamed by looking at the chain's various basis states and how they're related to one another. This will all be doable if reverse engineering the given code from the article is not too difficult.

Week 13-15: Kogut and Susskind

The last three weeks' reading will be centered around the article *Hamiltonian formulation of Wilson's lattice gauge field theories* by John Kogut and Leonard Susskind. This paper is 14 very thick pages that begin with the definition of unit vectors on the lattice, jump into fermions on the lattice and their gauge-invariant space of states, and sprint through to the lattice's gauge-field Hamiltonian and how energy works. What should be covered in this section is uncertain, but perhaps a write-up of the paper in which a paragraph or so summary recapitulating each section's argument would do. From there, presumably covering the last two weeks, what is to be done is uncertain.