Quiz 2: Solutions

1. Find the disk of convergence for the series

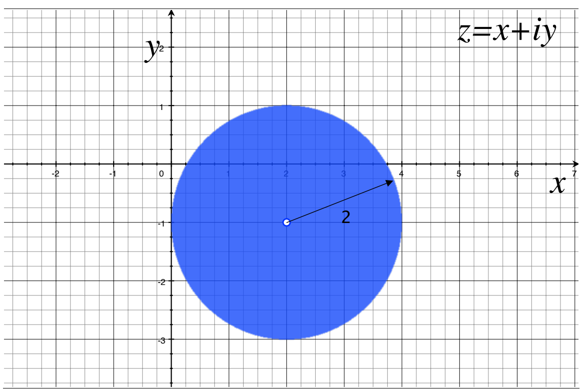
$$\sum_{n=0}^{\infty} \frac{(z-2+i)^n}{2^n}.$$

Draw this disk in the complex z-plane. Indicate its center with a dot and its radius on the drawing.

We use the ratio test and find

$$\rho_n = \left| \frac{(z-2+i)^{n+1}}{2^{n+1}} \frac{2^n}{(z-2+i)^n} \right| = \left| \frac{(z-2+i)}{2} \right| = \frac{|(z-2+i)|}{2},$$

which is independent of n so that $\rho = \lim_{n\to\infty} \rho_n = \rho_n$. Then the series converges if $\rho < 1$ and this gives $|(z-2+i)| < 2$. This is a disk of radius 2 centered at the complex point $z_0 = 2 - i$.



2. Find

$$2^i$$

in the form $x+iy$. No need to use a calculator. You can leave your answer with trigonometric and logarithm functions unevaluated.

To compute this complex power we need to use the complex logarithm, first note that

$$2^i = e^{\ln 2^i} = e^{i \ln 2},$$

where the second equality uses the property of logarithms that $\ln a^b = b \ln a$. Now, we can use Euler's formula to write

$$2^i = e^{i \ln 2} = \cos(\ln 2) + i \sin(\ln 2),$$

which is in $x + iy$ form.