Homework 1

Due Sunday, February 3rd at 5pm

Reading: Please read the excerpt "The Hidden Value of Ignorance" from B. Carey's book *How We Learn* before Wednesday:

faculty.bard.edu/ hhaggard/teaching/phys222Sp19/notes/CareyCh5.pdf.

Read Boas Ch. 3, sections 4 and 5. All the problems from Boas below refer to Chapter 3.

1. (a) Find a vector perpendicular to both $\hat{x}+\hat{y}$ and $\hat{x}-2\hat{z}$. (b) Square $(\vec{a}+\vec{b})$; interpret your result geometrically. [Hint: Your answer is a law that you learned in trigonometry.] (c) What is the value of $(\vec{A}\times\vec{B})^2+(\vec{A}\cdot\vec{B})^2$? [Hint: It's easier not to work in components for this one and your result is true in any number of dimensions. Comment: This is a special case of a more general identity, called Lagrange's identity, that we'll use more later in the semester.]

2. Boas Ch. 3, Sec. 5, Problem 1. I'll abbreviate this as Problem 5.1 from now on.

3. Boas Problem 5.45.

4. Boas Problem 5.24.

5. Without looking at a resource, invent your own proof that the magnitude of the cross product vector $\vec{c}=\vec{a}\times\vec{b}$ is equal to the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

6. Prove that the space of all bivectors in $\mathbb{R}^3$, which we name $\bigwedge^2\mathbb{R}^3$, is a vector space. As we discussed in class you can think of this space as the set of all linear combinations of the basis bivectors $\{\mathbf{e}_1\wedge\mathbf{e}_2, \mathbf{e}_1\wedge\mathbf{e}_3, \mathbf{e}_2\wedge\mathbf{e}_3\}$. I probably won't do this to you again this semester, but for this first attempt please check all of the vector space axioms. [For reference, the vector space axioms appear on p 179 of Boas. If you feel like you need more information on vector spaces you could read over sections 3.7,3.8, 3.10, and 3.14 too.]

7. (**a**) Let $\vec{a}=a_1\mathbf{e}_1+a_2\mathbf{e}_2+a_3\mathbf{e}_3$ and $\vec{b}=b_1\mathbf{e}_1+b_2\mathbf{e}_2+b_3\mathbf{e}_3$. Express the bivector $\vec{a}\wedge\vec{b}$ in the bivector basis $\{\mathbf{e}_1\wedge\mathbf{e}_2, \mathbf{e}_3\wedge\mathbf{e}_1, \mathbf{e}_2\wedge\mathbf{e}_3\}$ by expanding out the wedge product.

   (**b**) Write out the determinant form of the cross product

$$\vec{a}\times\vec{b} = \det\begin{pmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{pmatrix}.$$

   (**c**) Compare your calculations from (**a**) and (**b**). Does the comparison make sense? In particular, what do you make of the relations with parallelograms we discussed in class?