Homework 3

Due Sunday, February 17th at 5pm

Read Boas Ch. 5.

1. Boas Ch. 6, 4.10. [Hint: It may be helpful to think of the integral as the antiderivative here. That is, think about what you can take the derivative of to get the integrand of the integral.]

2. Boas 6.6.2

3. Boas 6.6.6

4. Boas 6.6.9

5. Boas 6.6.10

6. Boas 6.6.17

7. Boas 6.10.15

8. In class we studied $\hat{r}$ and $\hat{\theta}$. We realized that these varied throughout the polar plane. As you know, this means that each of these unit vectors represents a vector field. Call the integral curves of these vector fields the r- and θ-coordinate curves respectively.

   (**a**) Find the equations for the r-coordinate curves for a fixed, but arbitrary θ. That is find $x(r,\theta)$ and $y(r,\theta)$ along the r-coordinate. Do this by solving the integral curve equations

   $$\frac{d\vec{r}}{dr} = \hat{r} \qquad \text{or in components} \qquad \frac{dx}{dr} = \hat{r}\cdot\hat{x} \quad \text{and} \quad \frac{dy}{dr} = \hat{r}\cdot\hat{y},$$

   and using our result $\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y}$. To fix the integration constants you will need initial conditions, use the fact that you know that at $r = 0$ we have $x = y = 0$.

   (**b**) There was probably no surprise for you in part (**a**). Let's do this in a coordinate system you have probably never worked with. Let's call the two coordinates of this coordinate system σ and τ. The basis vectors of the coordinate system are given by

   $$\hat{\sigma} = \tau\hat{x} - \sigma\hat{y} \qquad \text{and} \qquad \hat{\tau} = \sigma\hat{x} + \tau\hat{y}.$$

   Find the σ-coordinate curves (for constant τ) using the equations

   $$\frac{dx}{d\sigma} = \hat{\sigma}\cdot\hat{x} \qquad \text{and} \qquad \frac{dy}{d\sigma} = \hat{\sigma}\cdot\hat{y}.$$

   Repeat the other way around for the τ-coordinate curves. Fixing your integration constants is a bit subtle. For both sets of differential equations use the initial conditions $x(\sigma = 0) = 0$. Fix the constants in the y equations by ensuring that the two equations you got for the σ- and τ-curves agree with each other. Can you guess the name of this coordinate system?