Math Methods II

Day 2

Today
0. Discussion of "Hidden Value of Ignorance"
I Last time
II Angular momentum & cross products
III 3D geometry with vectors

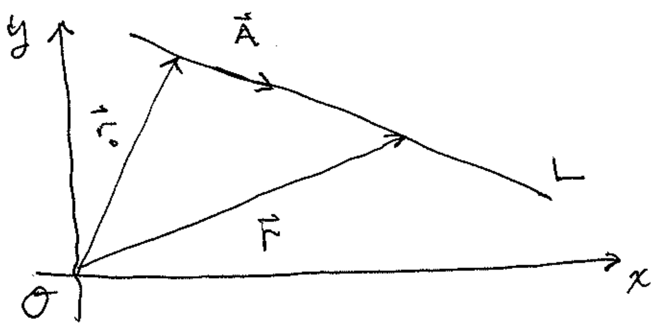
I. • Reviewed vector space axioms

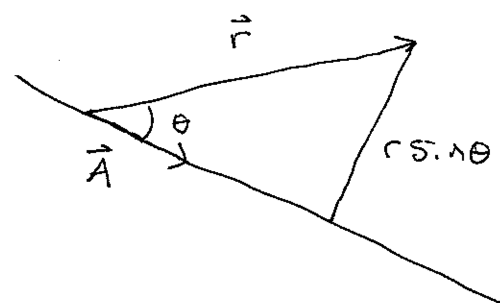
• Found 3 forms for the eqn. of a line:

$\vec{r} = \vec{r}_0 + t\vec{A}$; $\vec{A} = (a_1, a_2)$. (vec.)

$y = mx + b$; $m = \frac{a_2}{a_1}$, $b = y_0 - \frac{a_2}{a_1}x_0$. (Slope - Intercept)

$\frac{x - x_0}{a_1} = \frac{y - y_0}{a_2}$ (symmetric)





II What is the angular momentum of the ball w.r.t. O?

(1) $\vec{l} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m(\vec{r} \times \vec{v})$

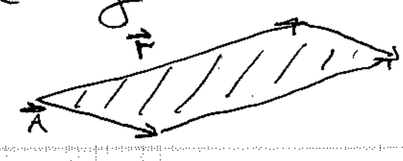
We noted last time $\vec{v} = \vec{A}$ (see vec.), so

$\vec{l} = m(\vec{r} \times \vec{A}) \Rightarrow l = mrA\sin\theta$

l = mass × speed × perp. distance from O to L.

$\hat{l}$ = into page (use right-hand rule)

(2) l = area of parallelogram spanned by $\vec{r}$ and $\vec{A}$ × mass



(3) Descriptions (1) and (2) are good for interpretation. A good method for computation in components is

$$\vec{l} = m \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & z \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$= m\Big[(a_3 y - a_2 z)\,\hat{x} + (a_1 z - a_3 x)\,\hat{y} + (a_2 x - a_1 y)\,\hat{z}\Big].$$

Given vectors $\vec{u}, \vec{v}, \vec{w} \in V$ their <u>span</u> is the set of all vectors that can be obtained as linear combos of these vectors.

A basis is a set of linearly independent vectors s.t.

$$\text{Span}\{\hat{e}_1, \ldots, \hat{e}_N\} = V.$$

The dimension of a vector space is

$$\dim V = \#\{\hat{e}_1, \ldots, \hat{e}_N\} \; (= N \text{ here}).$$

Aside: <u>Vector space structure</u>

Inner product: $\vec{A} \cdot \vec{B} = \sum_{i=1}^{\dim V} A_i B_i$

Norm: $A = \sqrt{\vec{A} \cdot \vec{A}} = \|\vec{A}\| = |\vec{A}|$

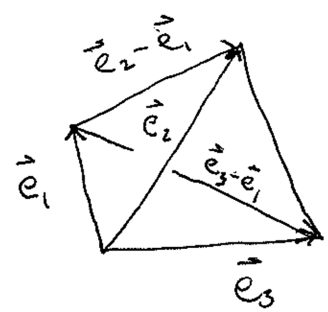
Cross product: $\vec{C} = \vec{A} \times \vec{B} = AB \sin\theta \, \hat{C}$ right hand rule

Basis: $\{\hat{e}_1, \hat{e}_2, \ldots, \hat{e}_N\}$

Orthonormal iff $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$ Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

III In quantum gravity a classical model for a grain of space is a tetrahedron.



Suppose you are given 3 of its edge vectors. Construct the 4 vectors perpendicular to its faces