Day 2 Feb 3rd, 2021

I Last time
II. The 'And' operation
III The 'Or' operation
IV Bayes Theorem ←

I Probability w/ conditions

$$P(A \mid B) = P_B(A)$$

conditioning

e.g. Prob. of an Ace card given that the card has a red suit.

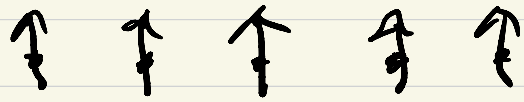
or

Prob. of drawing a red Skittle from a bowl of Skittles and M&Ms.

- Always identify your Sample Space.

II 'And'

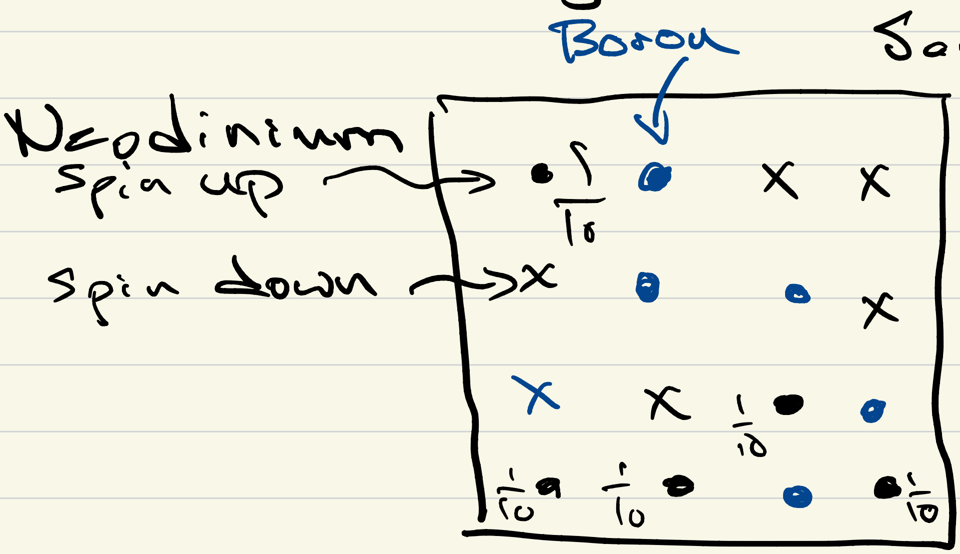
Let's think of magnets:



'ferromagnet'

2D ferromagnet

Sample Space



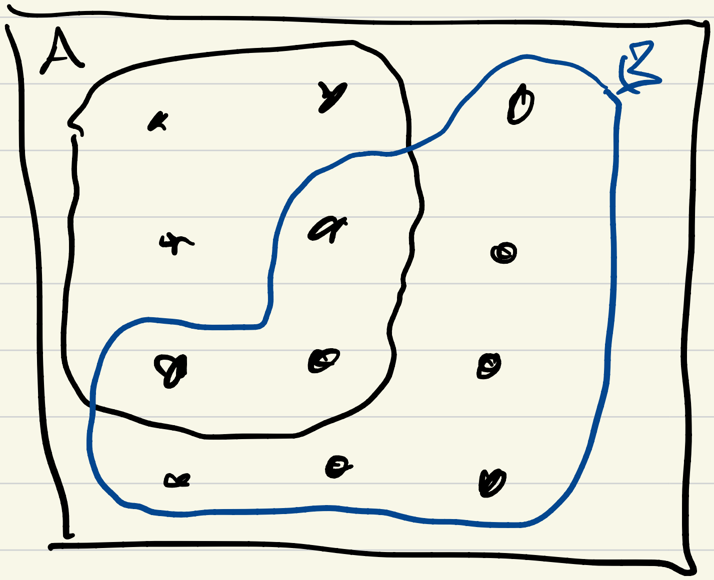
$P(\text{Boron}) = \frac{6}{16} = \frac{3}{8}$

$P(\text{Neo}) = \frac{5}{10} = \frac{1}{2}$

$P(\text{Boron and spin up}) = \frac{5}{16}$

Notice that up to now we've always assumed a uniform sample space w/ each event equally likely.

For now, assume uniformity.



$$P(A \text{ and } B) \equiv P(AB) = \frac{N(AB)}{N}$$

$$P(A) = \frac{N_A}{N} \; ; \; P(B) = \frac{N_B}{N}$$

$$P(B|A) = P_A(B) = \frac{3}{6} = \frac{N(AB)}{N_A}$$

$$\neq$$

$$P(A|B) = P_B(A) = \frac{3}{9} = \frac{N(AB)}{N_B}$$

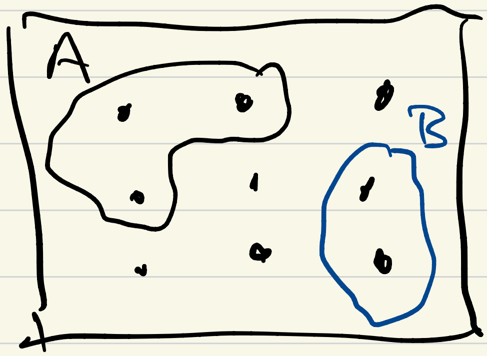
$$P(AB) = P_A(B) \cdot P(A) \qquad !$$

$$= P_B(A) \cdot P(B)$$

$$= P(BA)$$

When is it true that

$$P(AB) = P(A) \cdot P(B) ?$$



Not indep!

$$P(A) = \frac{1}{3} \qquad P(B) = \frac{2}{9}$$

$$P(AB) = 0$$

Only when events are <u>independent</u> does 'And' reduce to a simple product:

$$P(AB) = P(A) \cdot P(B)$$

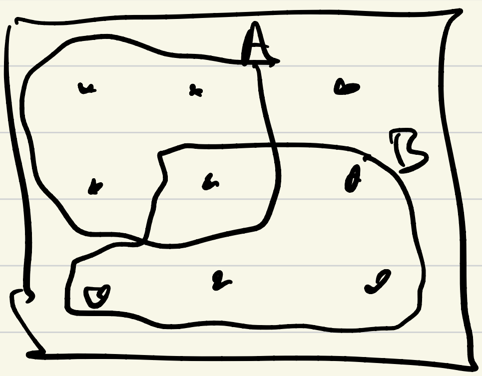
<u>Flip coin 2 times</u>

HH HT TH TT

Indep. Events.

III 'Or'

$P(A \text{ or } B)$ = probability that either A holds or B holds.



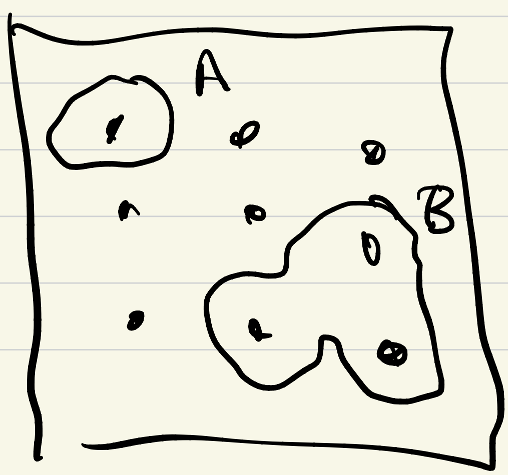
$$P(A+B) = \frac{8}{9} = \frac{4}{9} + \frac{5}{9} - \frac{1}{9}$$

$$= P(A) + P(B) - P(AB)$$

When does 'or' simplify?

$P(A+B) = P(A) + P(B)$

'Mutually exclusive events'



IV Bayes' Theorem

$$P(AB) = P_A(B) \cdot P(A)$$

$$= P_B(A) \cdot P(B)$$

$$= P(BA)$$

$$P_B(A) = P(A|B)$$

$$= \frac{P_A(B) \cdot P(A)}{P(B)}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)}$$

<u>Ex.</u> Power of Bayes for science:

$$P(\text{hypothesis} \mid \text{data})$$

$$= \frac{P(\text{data} \mid \text{hypothesis}) \cdot P(\text{hypothesis})}{P(\text{data})}$$