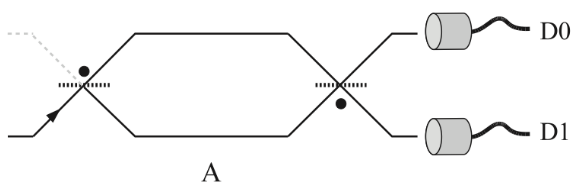
Homework 12

Due May 5th, 2017 at 5pm

Chapter 4 of Fox's book *Quantum Optics: An Introduction.* You can download this sample chapter by clicking here.

1. Consider the simplified Mach-Zehnder interferrometer shown below. Introduce a photon along



the lower beam, so that the input amplitude vector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Make a table of the possible outcomes and their probabilities for this setup.

2. Using the setup of the last problem, now suppose that you put a photographic plate at the point A in the lower beam. Once again make a table of possible outcomes (now there are three) and their probabilities for this setup.

3. Avshalom Elitzur and Lev Vaidman (1993) used the setup of the last two problems in a creative and surprising way. They imagined a factory that produces a type of firework triggered by light.[1] So sensitive is the trigger that the passage of a single photon through its mechanism will explode a firework.

   Because of manufacturing defects, however, many fireworks come off the assembly line without working triggers. Photons pass through these mechanisms without being registered at all, and the fireworks are duds. The factory managers want to be able tell for sure that at least some fireworks are in working order. How can they do this? Of course, if they send a photon through a given firework, and it blows up, then they can be sure that the firework was in working order—but they have also destroyed that firework. What the managers want is a way to identify fireworks that are explosive, but are not yet exploded. Since the firework triggers are set off even by one photon, this appears impossible.

   (**a**) Explain how the interferometer of the last two problems can be used to do the job.

   (**b**) Suppose the interferometer test is performed on a large number of fireworks from the factory. When the test is inconclusive on a particular firework, it is repeated until the fireworks

[1] They actually used bombs in their thought experiment. I prefer the less violent, but equally dramatic firework.

status is reasonably certain. What fraction of the working fireworks are certified as working but not detonated?

4. In class I mistakenly set $c = 1$ like a true theorist. In this problem I will, amongst other things, ask you to sort out where the factors of c should be in some of our expressions. You can check units to figure this out.

(**a**) In class I proved that

$$\langle E_n \rangle = \frac{-\frac{d}{d\beta}\left(\frac{1}{1-e^{-\hbar\omega_n\beta}}\right)}{\frac{1}{1-e^{-\hbar\omega_n\beta}}},$$

where $\beta = k_B T$. However, I skipped the calculus and algebra necessary to show that this gives

$$\langle E_n \rangle = \frac{\hbar\omega_n}{e^{\hbar\omega_n\beta} - 1}.$$

Prove that this is the case. Are there any factors of c missing in these expressions?

(**b**) I then went on to show that in the continuum limit $L \to \infty$

$$E(\omega) = 4\pi\hbar\frac{L^3}{8\pi^3}\int_0^\omega d\omega' \frac{\omega'^3}{e^{\hbar\omega'\beta} - 1}.$$

Are there factors of c missing in this expression or not? If so, insert the proper power of c to give the formula in SI units.

(**c**) Finally, I went on to derive $\mathcal{I}(\omega)$ the irradiance density as a function of ω. Redo this derivation with the correct factors of c in the starting formula and convince yourself that you get Planck's result for the blackbody spectrum.