Homework 4

Due March 3rd, 2017 at 5pm

Reading Hecht Ch. 9.

1. Hecht 2.39

2. In class I claimed that the superposition of two harmonic waves of the same frequency ω was accurately represented by the addition of *phasors*. Let's prove this. Suppose you have two harmonic waves that you choose to represent with complex exponentials

$$E_1 = E_{01}e^{i(\alpha_1+\omega t)} \qquad \text{and} \qquad E_2 = E_{02}e^{i(\alpha_2+\omega t)}.$$

We would like to represent their superposition $E = E_1 + E_2$ in this same form, that is, as

$$E = E_0 e^{i(\alpha+\omega t)},$$

and want to know how E_0 and α relate to E_{01}, E_{02}, α_1, and α_2.

(**a**) First proceed algebraically. Find E_0 using the formula for the magnitude of a complex number $E_0^2 = EE^*$. Find α using the real and imaginary parts of E. [Hint: For this part it is of value to cancel of the ωt dependence before you start solving for α.] This establishes the results we want to get out of any geometrical procedure we use for adding phasors.

(**b**) Using the geometry of phasor addition that we discussed in class find the magnitude and phase that results upon adding the two phasors E_1 and E_2. Are your results in agreement with part (**a**)?

3. (**a**) In class we used Maxwell's equations to show that the magnetic field satisfies the classical wave equation. Show that the electric field also satisfies the classical wave equation.

(**b**) A plane electric wave of the form

$$\vec{E} = E_o\hat{y}e^{i(kz-\omega t)}$$

moves through the vacuum. What is its direction of propagation? Use the Faraday and Ampère laws to find the accompanying magnetic field. [NB: Take care with partial differential equations. If you have a function $f = f(x, y)$ and the equation $\partial f/\partial x = 2x$ then this integrates to $x^2 + g(y)$, where g is an arbitrary *function* of y, not a constant.]

(**c**) For this wave find $\vec{E}\cdot\vec{B}$ for all times. Find $\vec{E}\times\vec{B}$. Verify the identity $|\vec{E}| = c|\vec{B}|$, which goes a long way to explaining why electricity is so much more familiar than magnetism.

4. The irradiance (or intensity) of an electromagnetic wave is a measure of the energy it can deposit on a detector. All practical detectors have a finite area and measure energy for finite intervals of time. So, in fact, the irradiance is the energy per unit area per unit time of the radiation. Thus once you know the area of your detector being hit by the wave and how long

it was lit you can find the total energy deposited from the irradiance I. In electromagnetism the irradiance is given by

$$I = \frac{1}{\mu_o}\langle|\vec{E}\times\vec{B}|\rangle_T,$$

where the angled brackets indicate a time average over a length of time T much longer than the period of the EM wave τ, that is, $T >> \tau$.

(**a**) Check that the units of irradiance are as I have claimed.

(**b**) Hecht 3.15.

(**c**) Hecht 3.18. For this one, you can use the harmonic wave from Problem ZZZ above if you like.

5. In section 2.9 of our text Hecht gives a beautiful derivation for the functional form of a spherical wave. Reread this argument. Suppose you had an electromagnetic spherical wave

$$\vec{E} = \frac{E_0}{r}\hat{\theta}e^{i(kr-\omega t)} \qquad \text{and} \qquad \vec{B} = \frac{B_0}{r}\hat{\phi}e^{i(kr-\omega t)}$$

moving out from a point source.

(**a**) Using your results from the last problem, what would the irradiance of the source be?

(**b**) Suppose this wave was radiated from the star at the center of the Trappist-1 System and traveled past Trappist-1c, about 0.015AU from the star, and on to Trappist-1g, about 0.045AU from the star. Which of the aliens living on each of these planets would see more radiant power and by what factor? Is this a familiar scaling?

(**c**) What energy would you find if you setup a detector to collect the radiated energy from the point source on a sphere of radius R and over a time T (a symbolic answer is fine)? How does this compare with the total energy radiated by the point source in that same time?