Homework 8

Due April 7th, 2017 at 5pm

Reading Hecht Ch. 10.1-10.2.5.

1. (**a**) In class we found the total reflected wave in the context of light incident on a thin film and focused by a lens to a point P. Taking the same setup, find the total transmitted wave and prove the result stated in class that

$$E_t = E_0 e^{-i\omega t}\left[\frac{tt'}{1-r^2e^{i\delta}}\right]e^{i\delta/2}.$$

(**b**) Under the assumption that the film has no absorption, find the transmitted intensity at P'. Simplify your result using the Stokes' connection formulae that we derived in class.

2. Taking up the context of Problem 1, relax the assumption that there is no absorption. Again let $T = tt'$ and $R = r^2$ be the transmission and reflection power coefficients and define the absorption power through $T + R = 1 - A$. Prove that

$$I_t^{\max} = I_0\left(1 - \frac{A}{1-R}\right)^2.$$

3. In class we defined the Finesse of a Fabry-Perot cavity:

$$\mathscr{F} = \frac{\text{FSR}}{\text{FWHM}} = \frac{\text{Free Spectral Range}}{\text{Full Width at Half Maximum}}.$$

Using your result from Problem 2 and the small angle approximation, show that the Finesse of the cavity is

$$\mathscr{F} = \frac{\pi\sqrt{R}}{1-R},$$

and hence only depends on the reflecting power of the cavity, even when there is absorption.

4. Fabry-Perot cavities are useful for spectroscopy. The full width at half maximum determines your ability to resolve closely-spaced frequencies in the source. Suppose you have a source with two frequency components, which you seek to resolve by sending the source through a Fabry-Perot cavity and varying the distance between the cavity mirrors. What is the smallest difference in frequency that you could detect using a cavity of finesse $\mathscr{F}$ and length d assuming the average value of the two frequencies is f_0? In other words, what is $\Delta f_{\min}/f_0$? [Hints: The conventional criterion is that the two peaks of the source are separated by at least a full width at half maximum. This is somewhat arbitrary, but common and you should use it for this problem. Feel free to take normal incidence $\theta_i = 0$, $n = 1$, and $\Delta f << f_0$.]

5. Hecht 9.54

6. Hecht 10.3

7. (**a**) Modeling a single slit as N sources each separated from its neighbors by a distance d, find the intensity of the central maximum in terms of I_0.

   (**b**) Qualitatively what happens to the central maximum as N increases? Why does this happen?

   (**c**) Plot the intensity relative to the single source intensity, I/I_0, for $N = 2$, $N = 4$, and $N = 10$.