Optics
Day 10

Today

I Last time

II Exam and guest lecture logistics

III Coherent & Incoherent Interference

IV The double slit

I
- Identified

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) = \epsilon_0 E^2$$

and

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

as the electromagnetic energy density and the power per unit area crossing a surface whose normal is $\hat{n} \parallel \hat{S}$, respectively.

- Found the irradiance or intensity

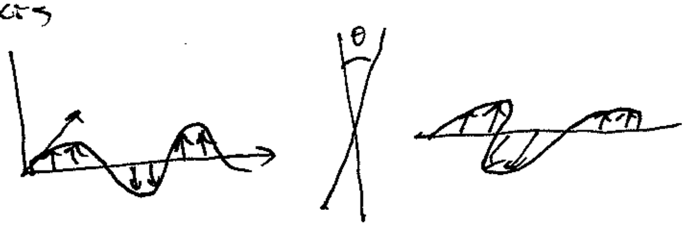
$$I = \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2$$

with $T \gg \tau$.

- Argued for Malus' law

$$I(\theta) = I(0) \cos^2\theta$$

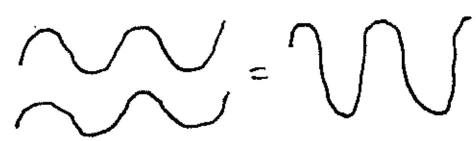
where θ is the tilt of the transmission axis



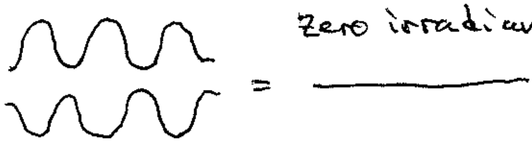
II I would like to hold our in-class exam not next week, but the week after on Thursday, March 16^{th}. A sign up sheet for guest lectures will be available soon.

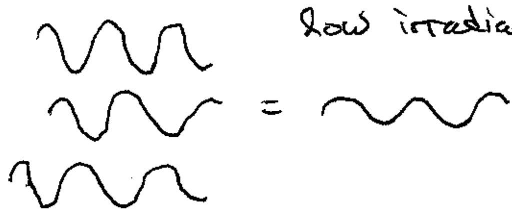
III <u>Variety of Superposition</u> <u>Irradiance</u>

In phase

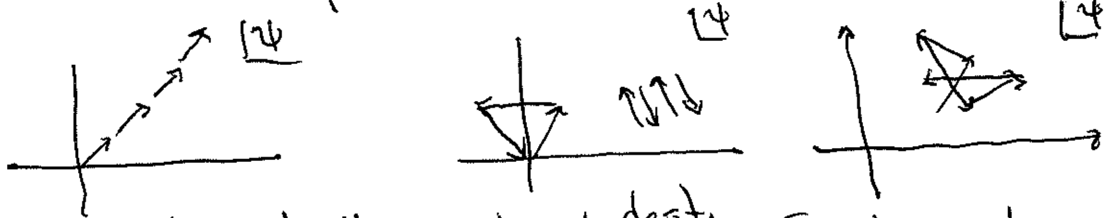


High "Constructive Interference"

180° out of phase  Zero irradiance — Destructive Interference

Many different phases  low irradiance — Incoherent addition

Phasor description



Coherent constructive Coherent dest. Incoherent

If we do this for a superposition of many waves, the $\cos\delta_{ij}$ tend to cancel out and

$$I_{tot} = I_1 + I_2 + \dots + I_n$$

This shows that incoherent superposition just adds irradiances.

IV Young brilliantly used a single slit to get a coherent source of light, which he then illuminated a double slit with.

Again for harmonic waves

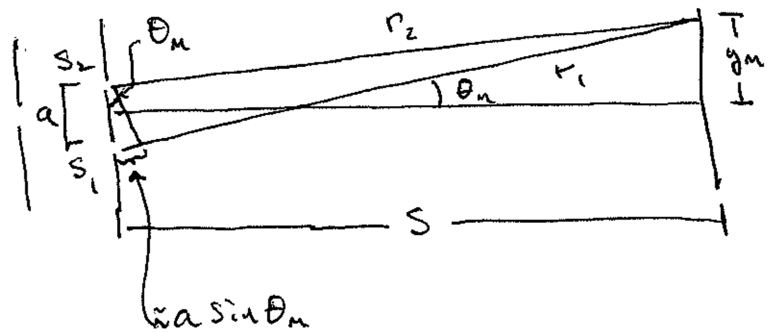
$$I = \tfrac{1}{2} c\epsilon_0 |\vec{E}_0|^2$$

When $\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ we have $|\vec{E}_0|^2 = \vec{E}\cdot\vec{E}^*$. Then the irradiance of the sum of two waves is

$$I = I_1 + I_2 + c\epsilon_0 \,\mathrm{Re}\{\vec{E}_1\cdot\vec{E}_2^*\}$$

or if $E_i = \sqrt{I_i}\, e^{+i\varphi_i}$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\underbrace{\cos(\varphi_2 - \varphi_1)}_{\cos\delta}.$$



For small angles r_1 and r_2 are nearly parallel and

$$r_1 - r_2 \approx a\sin\theta \approx a\theta$$

As well

$$\tan\theta \approx y/S \Rightarrow \theta \approx y/S$$

So that

$$r_1 - r_2 \approx \frac{a}{S} y$$

Then the condition for <u>constructive</u> interference is that the path length difference is an integer multiple of the wavelength

$$r_1 - r_2 = m\lambda$$

Thus

$$y_m \frac{a}{s} = m\lambda \quad \text{or} \quad \boxed{y_m = \frac{s}{a} m\lambda}$$

for $m = 0, \pm 1, \pm 2, \pm 3, \ldots$

Equivalently, we can express this in terms of the angle $\boxed{\theta_m = \frac{m\lambda}{a}}$

$$\delta = \frac{(r_1 - r_2)}{\lambda} \cdot 2\pi = \frac{2\pi}{\lambda}(r_1 - r_2) = k(r_1 - r_2).$$

Putting this into the intensity result

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

for the case $I_1 = I_2 = I_0$ gives

$$I = 2I_0(1 + \cos\delta)$$

$$= 4I_0\cos^2\frac{\delta}{2} = 4I_0\cos^2\left(\frac{k(r_1 - r_2)}{2}\right)$$

Combining these results with results of section III we can also find the intensity. The phase difference along r_1 and r_2 of the two waves is just the path length difference divided by the wavelength λ and then converted to radians by multiplying by 2π:

Once again

$$(r_1 - r_2) \approx \frac{a}{s} y$$

So that

$$\boxed{I = 4I_0\cos^2\left(\frac{y a \pi}{s\lambda}\right)}$$

Both from $y_m = \frac{s}{a} m\lambda$ and from here it is quick to check

$$\Delta y \approx \frac{s}{a}\lambda.$$