Today

I Last time

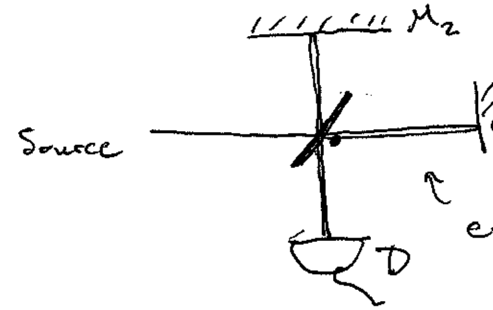
II Stokes connection between reflection & refraction

III Multiple Beam Interference and the Fabry-Perot interferometer

Optics
Day 12

Mar 9^{th}, 2017

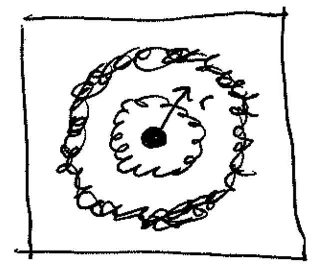
• Michelson Interferometer



excess OPL = d

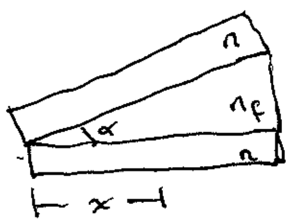
Found dark fringes at

$$2d\cos\theta_m = m\lambda$$



$\theta \approx \frac{r}{f}$

• Thin wedge interference



Found bright fringes at

$$x_m = \frac{(m+\frac{1}{2})}{2\alpha}\lambda_f$$

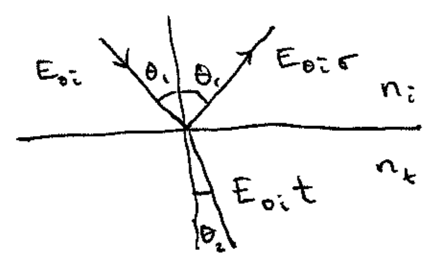
where $\lambda_f = \frac{\lambda_0}{n_f}$.

II Suppose we have a linearly polarized, plane electromagnetic wave

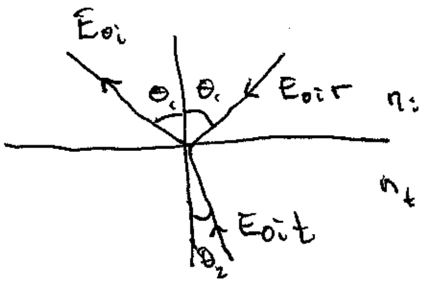
$$\vec{E}_i = \vec{E}_{0i}\, e^{[i(\vec{k}_i\cdot\vec{r} - \omega_i t)]}$$

Since it is linearly polarized $\vec{E}_{0i}$ does not depend on t. Let $E_{0i} = |\vec{E}_{0i}|$ and similarly for $\vec{E}_r$ and $\vec{E}_t$, then we can define

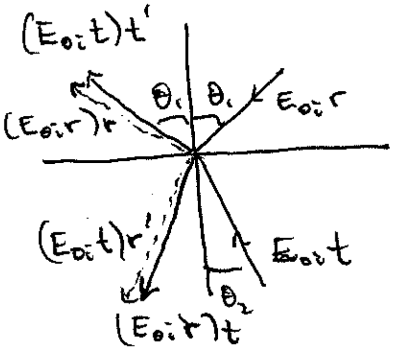
$$r \equiv \left(\frac{E_{0r}}{E_{0i}}\right);\ t \equiv \left(\frac{E_{0t}}{E_{0i}}\right)$$



Recall that when we studied Fermat's principle we found that every allowed path for light should also be allowed when traversed in reverse – the principle of reversability. Then another valid situation is



or



Nota Bene: We are <u>not</u> claiming that the r's and t's are independent of angle. So, these equations are really shorthand for

$$t(\theta_1)t'(\theta_2) = 1 - r^2(\theta_1)$$

and

$$r'(\theta_2) = -r(\theta_1)$$

where θ_1 and θ_2 are related by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

that is, $n_i \sin\theta_1 = n_t \sin\theta_2$.

But for both pictures to hold we must have

$$E_{0i}tt' + E_{0i}rr^* = E_{0i}$$

and

$$E_{0i}rt + E_{0i}tr' = 0$$

Here r' and t' are the amplitude coeffs when coming from the material with index n_t. Hence

$$tt' = 1 - r^2$$

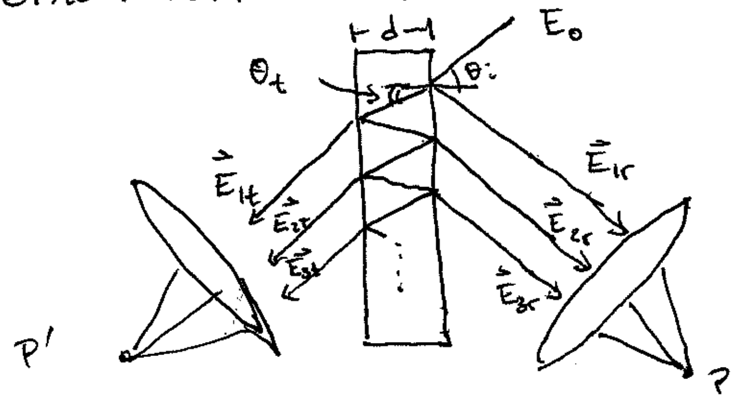
and

$$r' = -r$$

The minus sign in $r' = -r$ is precisely the π phase shift that we discussed for our half-sivered beam splitter.

We will consider detailed forms for the r's and t's soon, they are called the Fresnel equations.

III. So far we have only considered interference of two or three beams of coherent light, but this is too restrictive. Let's do <u>Multiple-beam inteference from a parallel film</u>: For a highly reflective film we should include all #'s of internal reflections:



So, $\Delta OPL = \overline{AC}\, n\left(\frac{1}{\sin\theta_t} - \sin\theta_t\right)$

$$= 2d\tan\theta_t\, n\left(\frac{1-\sin^2\theta_t}{\sin\theta_t}\right)$$

$$= 2dn\cos\theta_t. \quad \blacksquare$$

Continuing:

$$E_{3r} = E_0 t (r')^3 t' e^{-i\omega t} e^{2i\delta}$$

$$\vdots$$

$$E_{Nr} = E_0 t (r')^{(2N-3)} t' e^{-i\omega t} e^{(N-1)i\delta}$$

The total transmitted and reflected amplitudes are

$$\vec{E}_{0t} = \sum_i \vec{E}_{it} \quad \text{and} \quad \vec{E}_{0r} = \sum_i \vec{E}_{ir}.$$

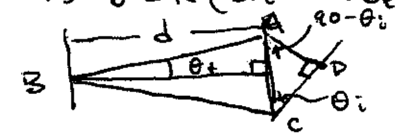
Using complex notation for the fields we have

$$E_{1r} = E_0 r e^{-i\omega t}$$ ← no need for spatial phase if we use this as our reference

$$E_{2r} = E_0 t r' t' e^{-i\omega t} e^{i\delta}$$ ← Now, extra phase is $\delta = k(2nd\cos\theta_t)$

<u>Pf</u>: $\Delta OPL = n\,\overline{ABC} - \overline{AD}$

$$= \overline{AC}\, n\left(\frac{1}{\sin\theta_t}\right) - \overline{AC}\sin\theta_i$$



Adding up all these contributions gives

$$E_r = E_0 e^{-i\omega t}\left[r + r't't e^{i\delta}\left\{1 + r'^2 e^{i\delta} + (r'^2 e^{i\delta})^2 + (r'^2 e^{i\delta})^3 + \ldots\right\}\right]$$

This is a geometric series and gives

$$E_r = E_0 e^{-i\omega t}\left[r + \frac{r'tt' e^{i\delta}}{1 - r'^2 e^{i\delta}}\right]$$

Finally using $r = -r'$ and $tt' = 1 - r^2$

$$E_r = E_0 e^{-i\omega t}\left[\frac{r(1-r^2e^{i\delta}) - r(1-r^2)e^{i\delta}}{1 - r^2 e^{i\delta}}\right] = \boxed{E_0 e^{-i\omega t}\left(\frac{r(1-e^{i\delta})}{1-r^2e^{i\delta}}\right)}$$