<u>Today</u>

Optics

Day 13

Mar 28th, 2017 P1/3

I Last time

II Return Exams & Discuss Them

III Multiple beam Interference Redux

I • Derived Stokes' connections:

$$tt' = 1 - r^2$$

and

$$r' = -r$$

Really,

$$t(\theta_1)t'(\theta_2) = 1 - r^2(\theta_1)$$

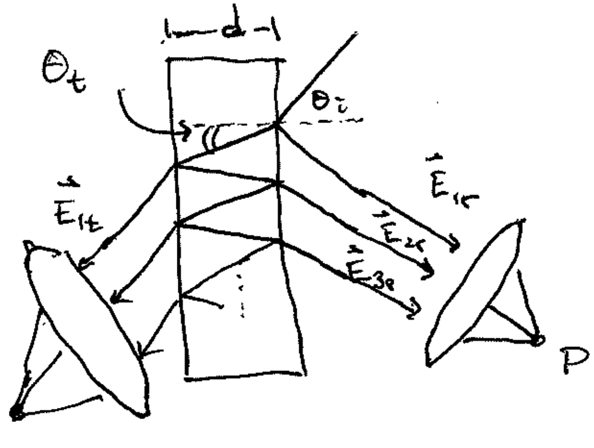
and

$$r'(\theta_2) = -r(\theta_1)$$

where

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

• Derived multiple beam interference



$$\vec{E}_{0r} = \sum_i \vec{E}_{ir}$$

and we found

$$E_r = E_0 e^{-i\omega t}\left[r + \frac{r'tt'e^{i\delta}}{1 - r'^2 e^{i\delta}}\right]$$

We showed $\delta = 2knd\cos\theta_t = \frac{4\pi n}{\lambda_0} d\cos\theta_t$

III If there is no absorption at the interfaces we can use Stokes connection and δ is real:

$$E_r = E_0 e^{-i\omega t}\left(\frac{r(1 - e^{i\delta})}{1 - r^2 e^{i\delta}}\right)$$

Then

$$I_r = \langle Re(E_r)^2 \rangle = \langle \tfrac{1}{4}(E_r + E_r^*)^2 \rangle$$

$$= \frac{1}{2} E_r E_r^*$$

$$\overset{alg.}{=} I_0 \left[\frac{2r^2 \overbrace{(1 - \cos\delta)}^{2\sin^2(\frac{\delta}{2})}}{(1 + r^4) - 2r^2\cos\delta}\right], \quad I_0 = \frac{E_0^2}{2}$$

You will show that

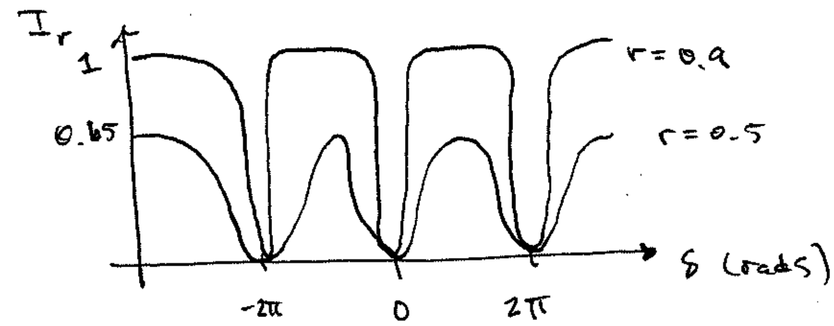
$$E_t = E_0 e^{-i\omega t}\left[\frac{tt'}{1-r^2 e^{i\delta}}\right] e^{i\delta/2}$$

and when there is no absorption

$$I_t = I_0 \frac{1}{1+\left(\frac{2r}{1-r^2}\right)^2 \sin^2 \delta/2}$$

In this case we can check

$$I_r + I_t = I_0$$



Can vary δ by changing the distance btwn reflective surfaces, the wavelength of the source, or the angle of incidence — this makes a very sensitive interferometer

Fabry-Perot Interferometer:

The same ideas and analysis apply when the thin film interfaces are coated with reflective (e.g. metallic) coatings. There are a few additional considerations

$$\delta = 2knd\cos\theta_t + 2\phi_r$$ e.g. from a phase shift due to the mirrors

and

$$tt' + r^2 = 1 - A$$ fractional power lost to absorption in mirrors

Then it is straightforward to show from our previous results, with

$T \equiv tt'$ transmission power

$R \equiv r^2$ reflection power

and $T + R = 1 - A$, that

$$I_t = I_0 \frac{T^2}{1+R^2-2R\cos\delta}$$

and

$$I_t^{max} = I_0 \frac{T^2}{(1-R)^2} = I_0 \frac{(1-R-A)^2}{(1-R)^2} = I_0\left(1-\frac{A}{1-R}\right)^2.$$

One measure of how good your Fabry-Perot cavity is for measuring interference fringes, is the

<u>Finesse</u>: A useful "figure of merit"

$$\mathcal{F} = \frac{FSR}{FWHM}$$

FSR = free Spectral Range
= distance btwn adjacent maxima

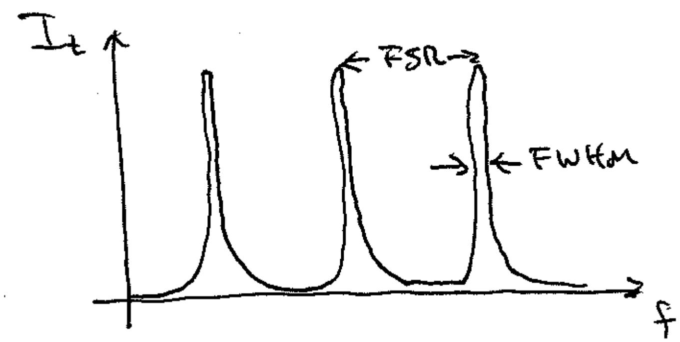
A good exercise is to show that

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1-R}$$

and depends only on the reflecting power of the mirrors.

FWHM = Full Width at Half Max
= width of a transmission maximum.

This is nice as it is a dimensionless ratio. But, we often measure it in the freq. domain



Note: with say $\theta_t = 0°$

$\delta = \frac{4\pi}{\lambda} d = 2kd$

and
$\omega = ck$, so

$\delta = \frac{2d}{c}\omega = 2\pi\left(\frac{2d}{c}\right)f$.