Optics

Day 14

March 30th, 2017 P1/3

Today

I Feedback Survey

II Last time

III Diffraction by slits

$\delta = 2knd\cos\theta_t + 2\phi$

is sensitive to changes in distance d, λ_0, and the angles θ_i, θ_t.

With absorption we let

$T = tt'$, $R = r^2$ and have $T + R = 1 - A$.

Then

$$I_t^{max} = I_0\left(1 - \frac{A}{1-R}\right)^2$$

- Introduced Finesse

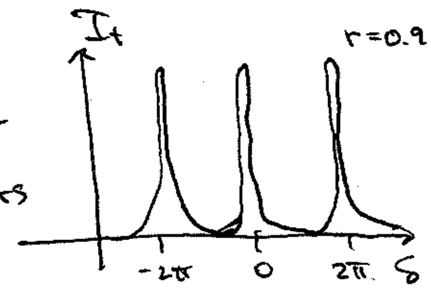
$$\mathcal{F} = \frac{FSR}{FWHM} = \frac{\text{dist. btwn maxima}}{\text{width of transition maxima}}.$$

I see feedback folder

II • Found

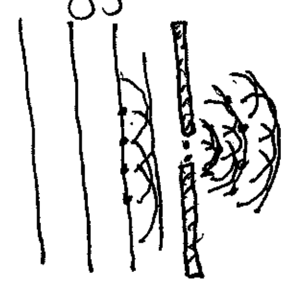
$$I_r = \tfrac{1}{2}E_r E_r^* = I_0\left[\frac{2r^2(1-\cos\delta)}{(1+r^4) - 2r^2\cos\delta}\right]$$

$$I_t = I_0\,\frac{1}{1+\left(\frac{2r}{1-r^2}\right)^2\sin^2\delta/2}$$



- Sharp transmission peaks make this a useful interferometer.

III On the very first day of class we introduced the notion of diffraction via Huygens' principle



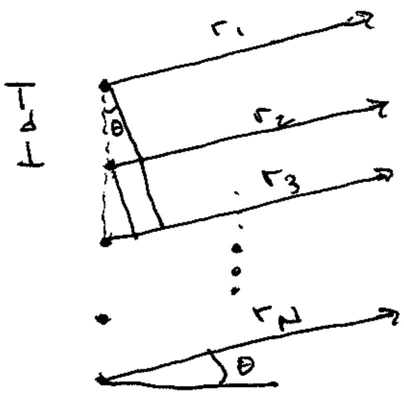
Wave fronts bend as they go through a slit

A better treatment of diffraction results when you include interference effects between the various wavelets. The resulting principle is the Huygens'-Fresnel principle:

- Each pt of a wavefront serves as a source of 2^{nd}-ary spherical wavelets
- The amplitude of the field is the superposition (interference) of all of the secondary wavelets.

We will be slowly building towards a rigorous justification of this principle

A simple model of this



$(r_2 - r_1) \approx d\sin\theta$

$(r_3 - r_1) \approx 2d\sin\theta$

$(r_N - r_1) \approx (N-1)d\sin\theta$

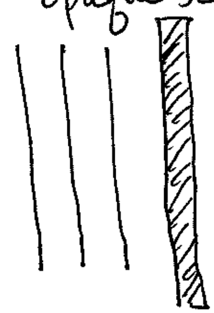
etc.

N total

So,

$$\tilde{E} = E_0(r)e^{-i\omega t}e^{ikr_1}\left[1 + e^{ik(r_2-r_1)} + \dots + e^{ik(r_N-r_1)}\right]$$

performed by Kirchoff. For now let's try to motivate it a bit.



Hence a slit is equivalent to

Now,

$$\delta = k(r_2 - r_1) = dk\sin\theta, \ \dots \text{etc.}$$

$$(N-1)\delta = k(r_N - r_1) = k(N-1)d\sin\theta$$

and so,

$$\tilde{E} = E_0(r)e^{-i\omega t}e^{ikr_1}\left[1 + e^{i\delta} + (e^{i\delta})^2 + \dots + (e^{i\delta})^{N-1}\right]$$

$$= E_0(r)e^{-i\omega t}e^{ikr_1}\left[\frac{e^{i\delta N} - 1}{e^{i\delta} - 1}\right]$$

$$= E_0(r)e^{-i\omega t}e^{ikr_1}\left[\frac{e^{iN\delta/2}\left(e^{iN\delta/2} - e^{-iN\delta/2}\right)}{e^{i\delta/2}\left(e^{i\delta/2} - e^{-i\delta/2}\right)}\right]$$

Then

$$\tilde{E} = E_0(r)\, e^{-i\omega t}\, e^{i[kr_1 + (N-1)\delta/2]} \left(\frac{\sin N\delta/2}{\sin \delta/2} \right)$$

but, the distance from the center of the slit to the screen is

$$R = \tfrac{1}{2}(N-1)\, d \sin\theta + r_1 ,$$

so

$$\tilde{E} = E_0(r)\, e^{i(kR - \omega t)} \left(\frac{\sin N\delta/2}{\sin \delta/2} \right)$$

and so

$$I = \tfrac{1}{2} \tilde{E}\tilde{E}^* = I_0 \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$$

An excellent exercise for yourself is to confirm that

$$\frac{\sin^2 N\delta/2}{\sin^2 \delta/2} = N^2 \quad \text{for } \delta = 2m\pi$$

and hence that at the principle maxima

$$\boxed{I = N^2 I_0.}$$

where I_0 is the intensity at P of a single one of the sources.

The interference pattern contains large "principal maxima" a smaller local maxima. The principle maxima are in directions such that $\delta = 2m\pi$, $m = 0, \pm1, \pm2, \ldots$ or when (since $\delta = kd\sin\theta$),

$$d \sin\theta_m = m\lambda, \quad m = 0, \pm1, \pm2, \ldots$$