Optics

Day 3

Feb 7^{th}, 2017

Today

I Last time

II Law of cosines & Taylor Series

III Spherical lenses

I

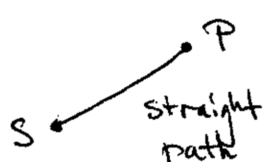
- Light in Media

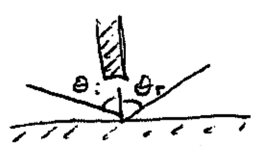
$$v = \frac{c}{n}, \quad \lambda = \frac{\lambda_0}{n}$$

- Fermat's Principle: In going from pt S to pt P light takes the path for which the traversal time is stationary w.r.t. neighboring paths.

- Used Fermat to derive

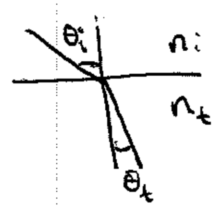
Light in 1 medium





Mirror reflection

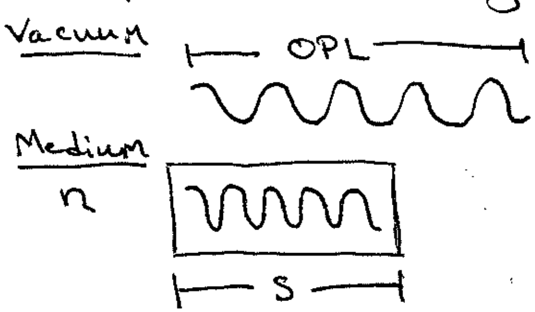
$\theta_i = \theta_r$



Snell:

$n_i s\theta_i = n_t s\theta_t$

- Optical Path Length (OPL)



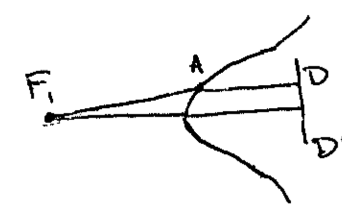
OPL answers the question: What distance would light have to travel in vacuum to have gone through as many wavelengths?

Means

$$\boxed{\frac{OPL}{\lambda_0} = \frac{s}{\lambda} \Rightarrow OPL = \frac{\lambda_0}{\lambda} s = ns}$$

More generally,

$$OPL = \int n(s)\, ds.$$

- 

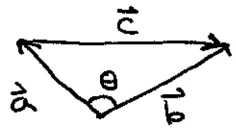
$$\underline{OPL}$$
$$n_i(\overline{F_i A}) + n_t(\overline{AD}) = \text{const.}$$

Hyperbola

Ellipses also have important lens properties. Collectively these are called aspherical lenses.

II Quick Reminders:

$\vec{c} = \vec{b} - \vec{a}$



$\Rightarrow \vec{c}^{\,2} = c^2 = (\vec{b} - \vec{a})^2$

$= a^2 + b^2 - 2\vec{a}\cdot\vec{b}$ Law of cosines

$\Rightarrow \boxed{c^2 = a^2 + b^2 - 2ab\cos\theta}$

Fourier Series

$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \cdots$

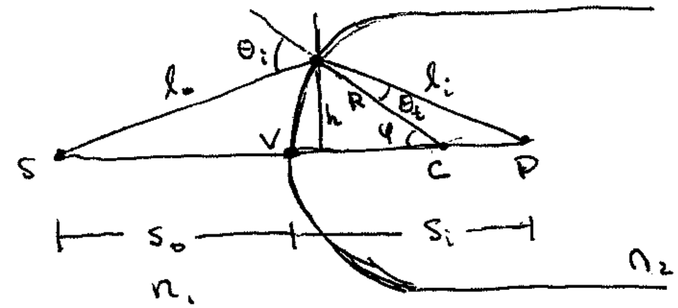
$\cos\varphi = \cos(0) - \sin(0)\,\varphi - \frac{1}{2}\cos(0)\,(\varphi)^2 + \cdots$

or

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} + \frac{\varphi^6}{6!} + \cdots$$

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \cdots$$

III A spherical interface



$OPL = n_1 l_o + n_2 l_i$

Using law of cosines

$$l_o = \left[(s_o+R)^2 + R^2 - 2R(s_o+R)\,c\varphi\right]^{1/2}$$

$$l_i = \left[(s_i-R)^2 + R^2 + 2R(s_i-R)\,c\varphi\right]^{1/2} \qquad c(\pi-\varphi) = -c\varphi$$

Apply Fermat's principle, varying A and hence φ, and require

$$\frac{d(OPL)}{d\varphi} = 0$$

$$= n_1\frac{dl_o}{d\varphi} + n_2\frac{dl_i}{d\varphi}$$

$$= \frac{n_1 R(s_o+R)s\varphi}{l_o} - \frac{n_2 R(s_i-R)s\varphi}{l_i}$$

$$\Rightarrow \frac{n_1 R^2}{l_o} + \frac{n_2 R^2}{l_i} = \frac{R n_2 s_i}{l_i} - \frac{R n_1 s_o}{l_o}$$

Finally then,

$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right).$$

Still a bit messy. The lengths l_i, l_o depend on φ $\rightsquigarrow$ not all rays from S go to same P!

However, for small φ, $\cos\varphi \approx 1$ and

$$l_o \approx s_o \quad \text{and} \quad l_i \approx s_i$$

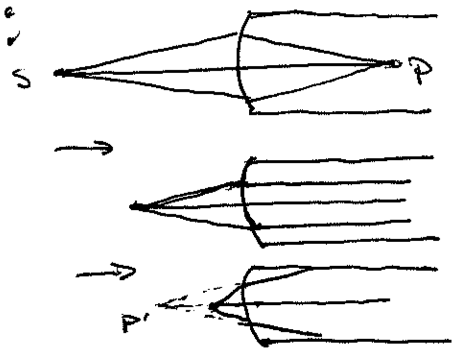
When we use this approx $\rightsquigarrow$ <u>First Order Theory</u>

Hence,

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}. \qquad (*)$$

Also called paraxial and Gaussian optics.

Consider what happens as we vary s_o:



We call $S = F_o$ the focus and

$$s_o = f_o = \frac{n_1 R}{n_2 - n_1}$$

the object focal length.

Similarly, if we send in parallel rays

then $s_o = \infty$ and we have an image focus $S_i = F_i$ and image focal

When s_o is s.t.

$$\frac{n_1}{s_o} = \frac{n_2 - n_1}{R}$$

s_i has to be ∞ to satisfy $(*)$.

This is the middle case at left.

When s_o continues to decrease, for $(*)$ to be satisfied the image must be to the left of the vertex V. We call this a <u>virtual image</u>.

At the boundary case, when $s_i = \infty$,

length

$$f_i = \frac{n_2 R}{n_2 - n_1}.$$

Sign Convention Table

Light Enters from left

s_o, f_o	+ left of V
s_i, f_i	+ right of V
R	+ if C is right of V