Feb 9th, 2017 1/3

Optics
Day 4

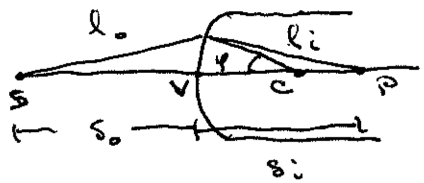
Today

I Last time

II Thin lenses

III Suggestions for ray tracing with thin lenses & Magnification

I • Derived the spherical surface result



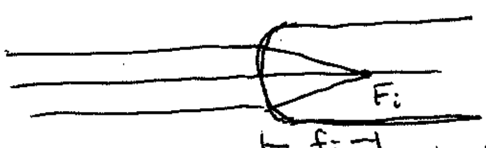
$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

• Took the paraxial approximation, which is small φ or $l_i \approx s_i$, $l_o \approx s_o$

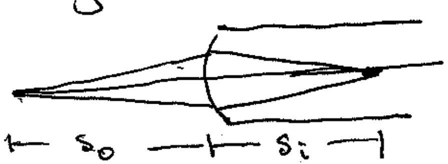
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

• Considered varying the object distance. Today let's vary the image distance

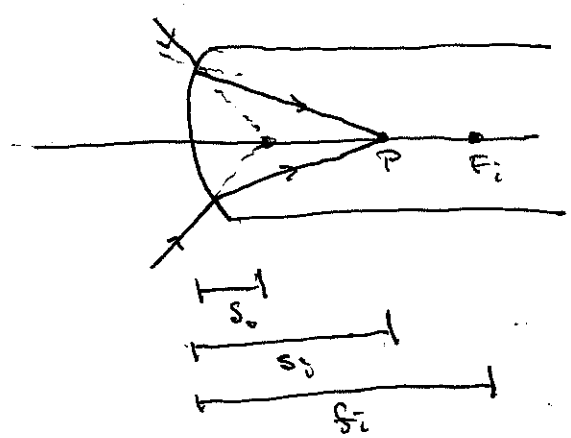
When $n_2/s_i = n_2 - n_1/R \Rightarrow s_o = \infty$



We call this image distance f_i the image focal length. For $s_i > f_i$ we have $s_o > 0$



But when $s_i < f_i$, $s_o < 0$ and we have



Sign Conv.s

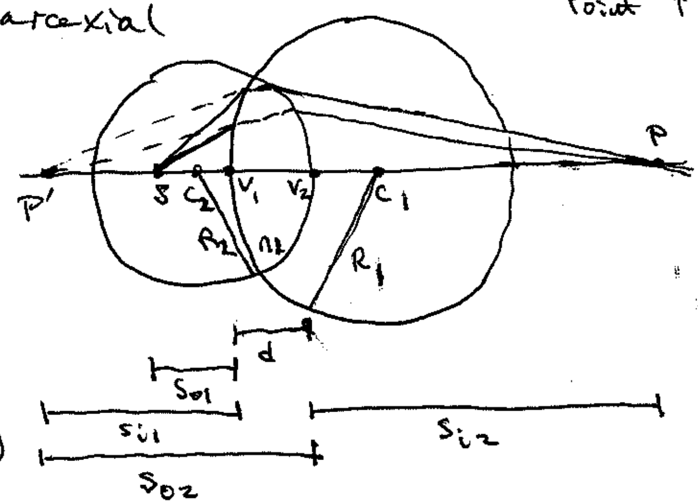
Light Enters @ Left	
s_o, f_o	+ left of V
s_i, f_i	+ right of V
R	+ if C is right of V

N.B: An object is "virtual" when rays converge toward it.

An image is "virtual" when rays diverge from it.

II An important application and further simplification of all of this is the case of thin lenses.

Strategy: Apply our paraxial result twice — once at the left interface and again at the right interface.



At left,

$$\frac{n_m}{s_{o1}} + \frac{n_\ell}{s_{i1}} = \frac{n_\ell - n_m}{R_1} \quad (1)$$

Note $n_\ell > n_m$ and $R_2 < 0 \Rightarrow \frac{n_m - n_\ell}{R_2} > 0$.

Taking (1) + (2) gives

$$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_\ell - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + n_\ell\left(\frac{1}{s_{i1} - d} - \frac{1}{s_{i1}}\right)$$

$$= (n_\ell - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n_\ell d}{s_{i1}(s_{i1} - d)}$$

In the limit $d \to 0$ this term is negligible.

Taking $n_m = n_{air} = 1$ and letting $s_{o1} = s_o$, $s_{i2} = s_i$ we have the <u>Thin Lens Eqn</u>:

For the second application note that

$$|s_{o2}| = |s_{i1}| + d$$

Point P' is left of V_2 so $|s_{o2}| = s_{o2}$, while $|s_{i1}| = -s_{i1}$ (L of V_1)

So,

$$s_{o2} = -s_{i1} + d,$$

and

$$\frac{n_\ell}{(-s_{i1} + d)} + \frac{n_m}{s_{i2}} = \frac{n_m - n_\ell}{R_2} \quad (2)$$

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Again we define

$$\lim_{s_o \to \infty} s_i = f_i \Rightarrow \frac{1}{f_i} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and

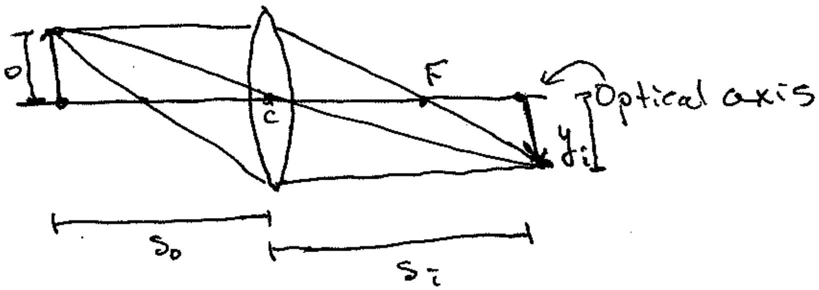
$$\lim_{s_i \to \infty} s_o = f_o \Rightarrow \frac{1}{f_o} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So,

$$\frac{1}{f_i} = \frac{1}{f_o} \Rightarrow f_i = f_o \overset{\text{defined as}}{\equiv} f$$

the "focal length" of the lens.

III So far we have only looked at imaging point sources. Let's consider the image of an extended source



There are 3 types of rays that we immediately know how to trace:

(i) Any ray through the center of the lens, pt C, continues on undeflected. In the diagram at left both the optical axis <u>and</u> the diagonal ray through C illustrate this.

(ii) Rays parallel to the optical axis are deflected through the focal pt F on the right.

(iii) Rays through the left focal point are deflected so that they are parallel to the optical axis.

Using these three rules we are able to find the image of the arrow tip (in fact of any pt. on the arrow) as illustrated. This shows that in this case the image is inverted.

The transverse magnification is defined by

$$\boxed{M_T = \frac{y_i}{y_o}}$$

Note that vertical distances below the optical axis are considered negative, this is a way to keep track of whether they are inverted or not.

Use similar triangles on the diagram to express M_T in terms of s_o & s_i alone.