Optics Day 6

Today

I Last time &

Magnifying glasses

II Compound Microscope

III The thick lens &

Aberration

- Defined three magnifications:

$$M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o} = -\frac{x_i}{f} = -\frac{f}{x_o}$$

The "transverse" magnification.

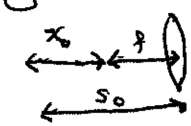
$$M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$

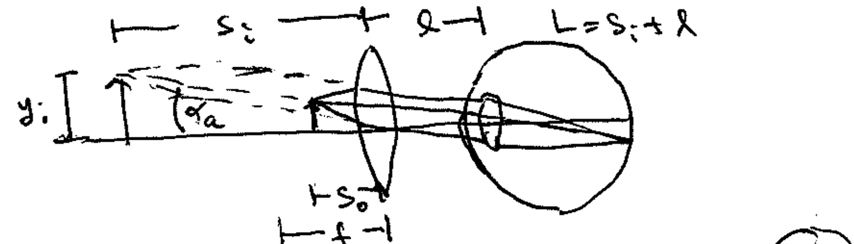
The "longitudinal" magnification.

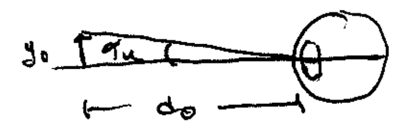
Finally we considered a magnifying glass

I
- Proved the center of lens rays go straight.
- Argued why thin lenses have a focal plane
- Used similar triangles to find the Newtonian form of the thin lens eqn:

$$x_o x_i = f^2$$







$$MP = \frac{\alpha_a}{\alpha_u}$$

$\tan \alpha_a = y_i / L \approx \alpha_a$ and

$\tan \alpha_u = y_o / d_o \approx \alpha_u$, so ($d_o$: near pt distance)

$$MP = \frac{y_i d_o}{y_o L}$$

The "magnifying power" or "angular mag."

We have, using $\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \Rightarrow \frac{s_i}{s_o}=\frac{s_i}{f}-1$

$$MP = -\frac{s_i}{s_o}\frac{d_o}{L} = \left(1-\frac{s_i}{f}\right)\frac{d_o}{L}$$

The image distance is negative so $s_i = -(L-l)$ and $\underset{1/f}{\overset{\text{"power of magnifier"}}{\mathcal{D}}}$

$$MP = \frac{d_o}{L}\left[1+\mathcal{D}(L-l)\right]$$

Bigger l makes smaller MP so take $l=0$

$$[MP]_{l=0} = d_o\left[\frac{1}{L}+\mathcal{D}\right]$$

Now we see that the smaller the value of L the better. For a given person the best value is $L=d_o$ and then

$$[MP]_{\substack{l=0\\L=d_o}} = 1+d_o\mathcal{D}$$

More commonly we take $s_o=f$ so that $L=\infty$ and

$$[MP]_{L=\infty} = d_o\mathcal{D}$$

This is not much smaller than the maximum and because the rays come into your eye parallel you needn't strain at all.

To give a sense of scale Ethan and I measured our near points to be $11\text{cm} = 0.11\text{m}$. With a lens of

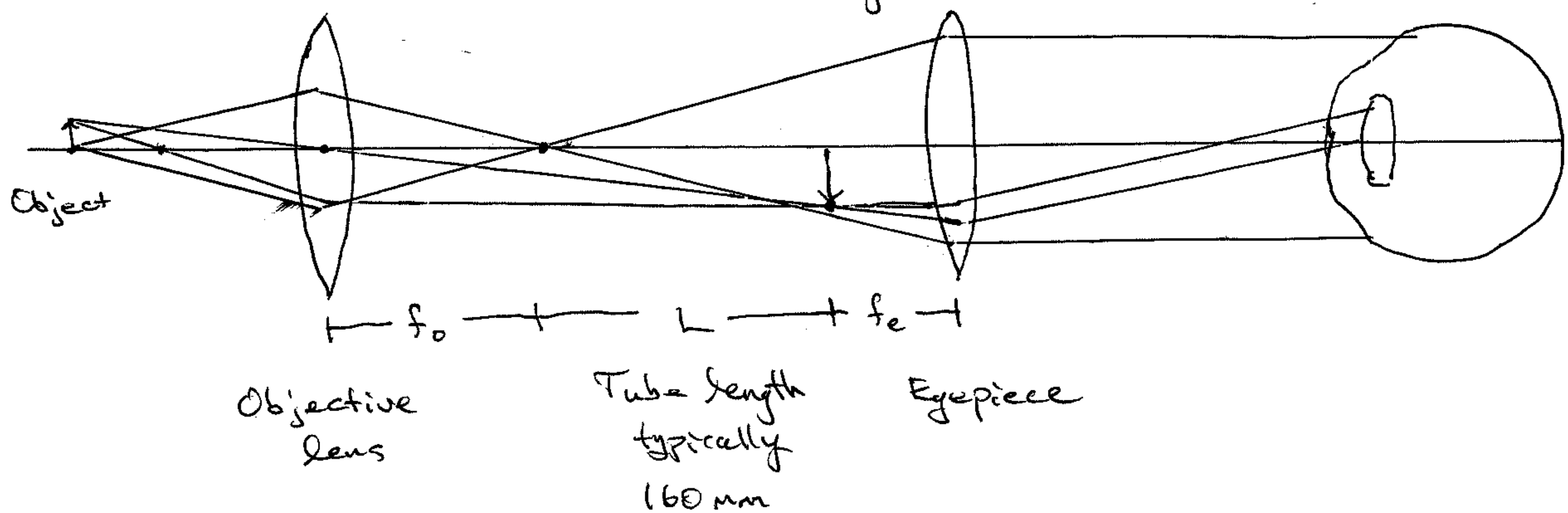
$$\mathcal{D} = 20 \text{ dioptres} = 20\,\frac{1}{\text{m}} \Rightarrow f = 5\text{cm}$$

We would get

$$MP = 2.2X.$$

II A compound microscope provides a simple instrument that can achieve $MP \sim 30X$.

The idea is to use a simple positive lens to achieve a transverse magnification and then to use a magnifying glass on this intermediate image to get another boost in magnification. The first lens is called the objective lens



and the second is the eyepiece.

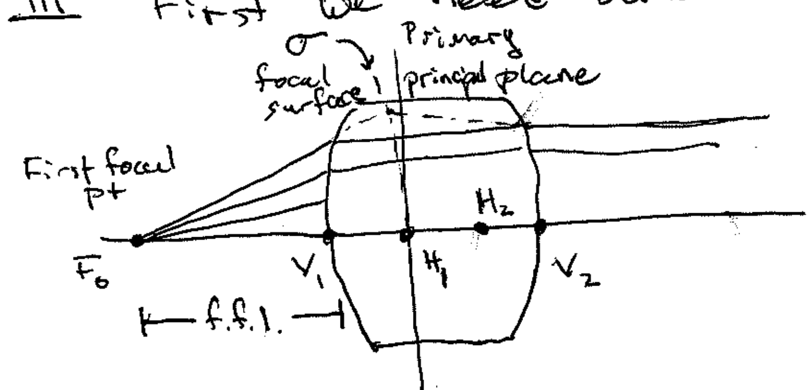
The magnification of this system is the product of the transverse magnification of the objective M_{To} and the angular mag of the eyepiece M_{Ae},

$$MP = M_{To} M_{Ae}$$

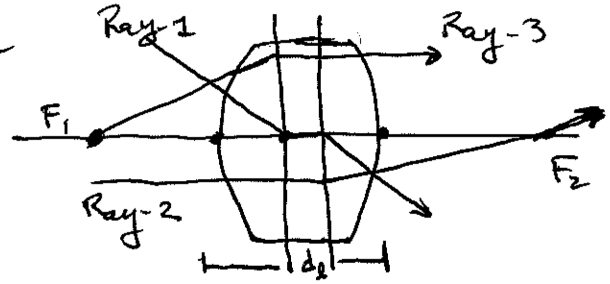
$$= \left(-\frac{16\text{cm}}{f_o}\right)\left(\frac{10-25\text{cm}}{f_e}\right)$$

$$\sim -(5)(10) = -50\times.$$

III First we need some definitions:



The pts H_1 and H_2 are determined by the primary and 2nd ary principal planes and are called the 1st and 2nd principal points. A modified version of ray tracing works for thick lenses where the lens is treated as a two piece system made up of its two principal planes. The three rays we know how to treat are



In fact,

$$\frac{1}{f} = (n_\ell - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_\ell - 1)d_\ell}{n_\ell R_1 R_2}\right]$$

and

$$\overline{V_1 H_1} \equiv h_1 = -\frac{f(n_\ell - 1)d_\ell}{R_2 n_\ell}$$

and

$$\overline{H_2 V_2} \equiv h_2 = -\frac{f(n_\ell - 1)d_\ell}{R_1 n_\ell}.$$

Wonderfully, using our previous analysis of two spherical interfaces and doing a lot of algebra you can find

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

where $s_o = \overline{PH_1}$ and $s_i = \overline{H_2 S}$ and the focal length is measured from the principal plane as well.