Optics
Day 7

Today

I Waves

II Complex numbers & phasors

I To date we have focused on a ray picture of light. However, several observed phenomena cannot be explained by this ray theory. It is time to turn to a description of light as a wave.

Hecht has a lovely argument leading from the physics of waves to the wave equation. A wave that doesn't change shape as it moves to the right is described by a function of the form

$$\psi_r(x,t) = f(x - vt)$$

as t evolves this term just moves the argument of f, hence shifting its values over.

Similarly, a wave moving to the left is described by

$$\psi_l(x,t) = g(x + vt)$$

In fact, the general solution to the wave equation

$$\frac{\partial^2 \psi}{\partial t^2} - v^2 \frac{\partial^2 \psi}{\partial x^2} = 0$$

is just a combination of these two

$$\psi(x,t) = \psi_r + \psi_l$$

Since

$$\frac{\partial^2}{\partial t^2}(\psi_r + \psi_l) - v^2 \frac{\partial^2}{\partial x^2}(\psi_r + \psi_l) = 0$$

as well (simply add the two eqns).
Important (!): called superposition.

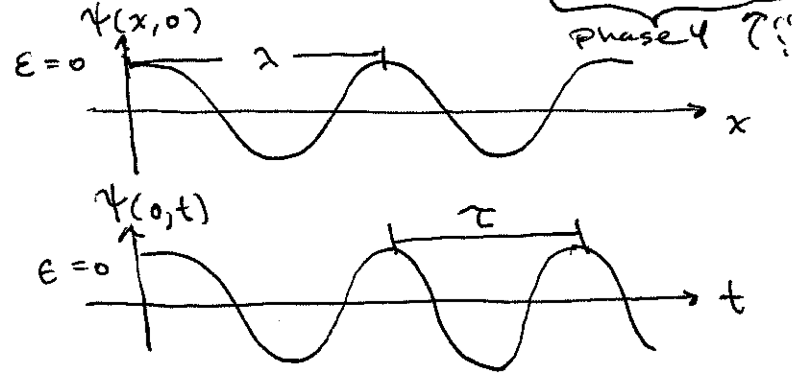
How many parameters (#'s) does it take to specify a wave then? Well, we need to fix f and g, which are general functions — this takes an infinite number of parameters! That's a mess. Are there waves specified by fewer parameters? If we fix a functional form, then the answer is yes! We choose as our base form the <u>harmonic waves</u>:

Harmonic waves are efficient — they are specified by just $\{A, \omega, k, \epsilon\}$ — but they are also quite useful; we will soon find that any periodic wave can be expressed as a sum of these solutions. But, we can manipulate these solutions even more efficiently using complex numbers.

II Introduce $i = \sqrt{-1}$ and the

$$\psi(x,t) = A\cos(\underbrace{\omega t - kx + \varepsilon}_{\text{phase } \varphi})$$

"initial phase" (ε)

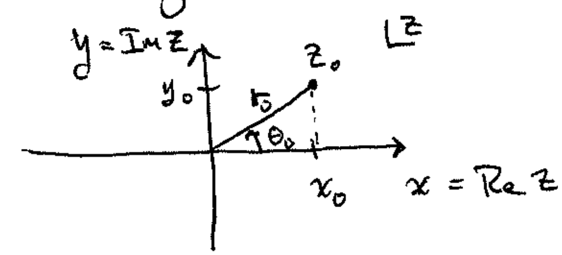


<u>Parameters</u>: A "amplitude"

$k = \frac{2\pi}{\lambda}$ "wave #" or $\lambda = \frac{2\pi}{k}$ "wavelength"

$\omega = \frac{2\pi}{\tau}$ "ang. freq." or $\tau = \frac{2\pi}{\omega}$ "period"

Combinations $z = x + iy$ with $x, y \in \mathbb{R}$. The complex z-plane or Argand diagram is



$r_0 = \sqrt{x_0^2 + y_0^2}$, $\theta_0 = \tan^{-1}(y_0/x_0)$

Wonderfully,

$$e^{i\theta} = \cos\theta + i\sin\theta$$

and so

$$re^{i\theta} = r\cos\theta + ir\sin\theta = x + iy = z$$

This establishes that we can encode harmonic waves in complex numbers

$$\psi(x,t) = A e^{i(\omega t - kx + \epsilon)} = A e^{i\varphi}$$

Conventionally we track

$$\mathrm{Re}\,\psi = A\cos(\omega t - kx + \epsilon),$$

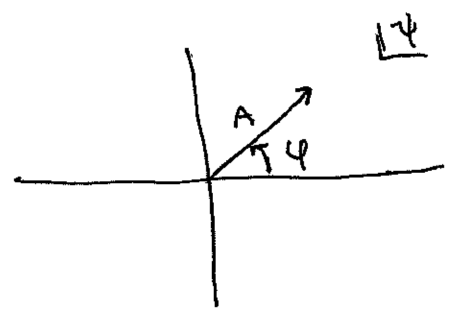
but $\mathrm{Im}\,\psi = A\sin(\omega t - kx + \epsilon)$ is also a valid harmonic wave.

Many operations do not mix these two waves: addition, subtraction, mult. by real #, division by real #, differentiation and integration w.r.t real variable.

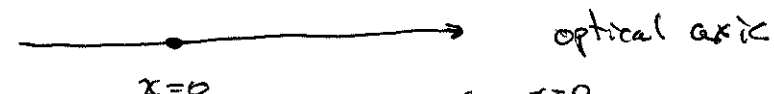
Complex mult. and division do not preserve the separate identities of these waves and careful consideration is required.

Phasors: A phasor encodes the parameters of the harmonic wave in a complex number and illustrates them graphically

$$\psi = A e^{i\varphi}$$



Consider the ray picture again



choose a point in space say x=0 and watch the wave at this point. Its phasor simply rotates