Optics

Day 9

Feb 28th, 2017

Today

I Last time

II Electromagnetic energy and the Poynting vector

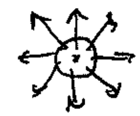
III Polarization & Coherence

Maxwell's Equations: In vacuum $\rho = 0$, $\vec{J} = 0$

$\oint\!\!\oint_A \vec{E} \cdot d\vec{S} = 0$ or $\vec{\nabla} \cdot \vec{E} = 0$, $\oint\!\!\oint_A \vec{B} \cdot d\vec{S} = 0$ or $\vec{\nabla} \cdot \vec{B} = 0$

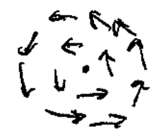
$\oint_C \vec{E} \cdot d\vec{l} = -\iint_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$ or $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, $\oint_C \vec{B} \cdot d\vec{l} = \frac{1}{c^2} \iint_A \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S} + \mu_0 \iint_A \vec{J} \cdot d\vec{S}$ or $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$

I
- Interpretted the divergence



$\vec{\nabla} \cdot \vec{E} > 0$

and the curl



$\vec{\nabla} \times \vec{B} > 0$

- Derived the wave equation

$$\frac{\partial^2 \vec{B}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \vec{B}.$$

II A quick way to access the energy contained in the electric field is to consider a parallel plate capacitor. The energy required to assemble q and $-q$ on the plates is $\frac{1}{2} C V^2$ where V is the voltage across the plates. This energy becomes stored in the electric field between the plates. This field fills a volume $A \cdot d$, where A is the plate area and d is their separation

Then

$$u_E = \frac{\frac{1}{2} C V^2}{A \cdot d} = \frac{\frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E d)^2}{A d} = \frac{1}{2} \epsilon_0 E^2$$

Similarly for a solenoid

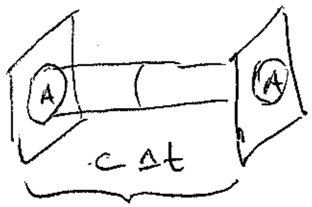
$$u_B = \frac{\frac{1}{2} L I^2}{A l} = \frac{\frac{1}{2}(\mu_0 n^2 l A)(B/\mu_0 n)^2}{A l}$$

$$= \frac{1}{2\mu_0} B^2$$

In general, $E = cB$, and so $u_E = u_B$ and when both are present

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

A propagating electromagnetic wave can impart its energy into a volume



$Ac\Delta t$

Then the energy carried across A per unit time is

$$S = \frac{u A c \Delta t}{A \Delta t} = uc$$

varies so quickly that practical devices average any measurement of S. This averaging defines <u>irradiance</u> or <u>intensity</u>

$$I = \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2$$

← show on homework.

Where $T >> \tau$ the period of the wave

<u>Poynting's Theorem</u>: Let's derive this more carefully and practice our vector derivatives. Suppose some charges and

Then

$$S = \frac{1}{\mu_0} c B^2 = \frac{1}{\mu_0} E B$$

This power is imparted by the wave in its direction of propogation. If we were to associate a direction to this propagation then

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.}$$

This is called the Poynting vector. At optical frequencies ($\approx 10^{15}$ Hz) this

Currents give rise to $\vec{E}$ and $\vec{B}$. The work done on q in time dt is

$$\vec{F} \cdot d\vec{l} = q(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{v}\, dt = q \vec{E} \cdot \vec{v}\, dt$$

With $q = \rho\, dV$ and $\rho \vec{v} = \vec{J}$ we have

$$\frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{J})\, dV$$

From Ampère's law

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu_0} \vec{E} \cdot (\vec{\nabla} \times \vec{B}) - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

and

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{B})$$

or

$$\vec{E}\cdot(\vec{\nabla}\times\vec{B}) = -\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

We have

$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\vec{\nabla}\cdot(\vec{E}\times\vec{B})$$

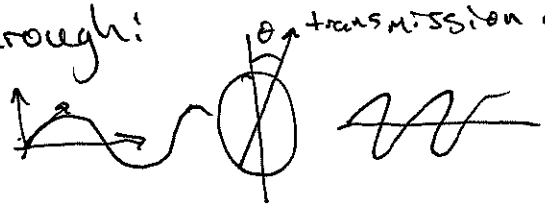
Then

$$\boxed{\frac{dW}{dt} = -\frac{d}{dt}\iiint_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)dV - \frac{1}{\mu_0}\oint_A (\vec{E}\times\vec{B})\, d\vec{S}}$$

The work done by the fields on q equals the decrease of energy in the fields and the energy that flows out through the boundary.

We call this direction the <u>polarization</u> of the electromagnetic wave.

A <u>polarizer</u> only lets the portion of the electric field along its axis through:



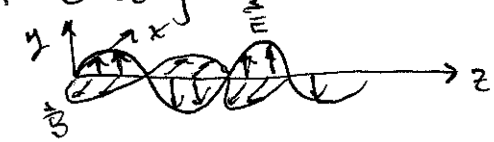
What happens to the irradiance in this process?

III <u>Irradiance & polarization:</u>

On the homework you show that EM waves are transverse, that is, both $\vec{E}$ & $\vec{B}$ are perp to the direction of propagation. For a harmonic EM wave of the form

$$\vec{E} = E_0\hat{y}\cos(kz-\omega t)$$
$$\vec{B} = -B_0\hat{x}\cos(kz-\omega t)$$

the direction of the E-field doesn't change in time



The transmitted magnitude is

$$|\vec{E}|_\theta = E_0\cos\theta\cos(kz-\omega t)$$

So,

$$I = \frac{c\epsilon_0}{2}E_0^2\cos^2\theta$$

$$\Rightarrow \boxed{I(\theta) = I(0)\cos^2\theta}$$

[stopped here.]

<u>Coherent & Incoherent Interference</u>

Varieties of <u>superposition</u>

In phase