Homework 2

Due Friday, September 18th in class

Finish Chapters 4 and 5 of Hartle's book. Beginning reading Hecht's Ch. 2. (I also encourage you to read Ch. 1, which is a very nice historical summary. However, this is not required.)

Problem 1 A particle is traveling at $\frac{3}{5}c$ in the x direction. Determine its proper velocity, η^μ (all four components).

Problem 2 Consider a collision in which particle A (with mass m_A and proper velocity η_A) hits particle B (mass m_B, proper velocity η_B), producing particle C (m_C, η_C) and particle D (m_D, η_D). Suppose that (relativistic) energy and momentum are conserved in system S (i.e., $p^\mu_A + p^\mu_B = p^\mu_C + p^\mu_D$). Using the Lorentz transformations, show that (relativistic) energy and momentum are also conserved in S'. (Do not assume that mass is conserved—in general, it is not: $m_A + m_B \neq m_C + m_D$.)

Problem 3 A pion traveling at speed v decays into a muon and a neutrino, $\pi^- \to \mu^- + \bar{\nu}_\mu$. If the neutrino emerges at 90° to the original pion direction, at what angle does the μ come off? [Answer: $\tan\theta = (1 - m^2_\mu/m^2_\pi)/(2\beta\gamma^2)$]

Problem 4 Particle A, at rest, decays into particles B and C ($A \to B + C$).

(a) Find the energy of the outgoing particles, in terms of the various masses.

$$\left[\textit{Answer:} \qquad E_B = \frac{m^2_A + m^2_B - m^2_C}{2m_A}c^2\right]$$

(b) Find the magnitudes of the outgoing momenta.

$$\left[\begin{array}{l}\textit{Answer:} \qquad |\vec{p}_B| = |\vec{p}_C| = \dfrac{\sqrt{\lambda(m^2_A, m^2_B, m^2_C)}}{2m_A}c, \\ \quad \text{where } \lambda \text{ is the so-called } \textit{triangle function}: \\ \qquad \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx.\end{array}\right]$$

(c) Note that λ factors: $\lambda(a^2, b^2, c^2) = (a + b + c)(a + b - c)(a - b + c)(a - b - c)$. Thus $|\vec{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary if $m_A < (m_B + m_C)$. Explain.

Problem 5 A 20-m pole is carried so fast in the direction of its length that it appears to be only 10m long in the laboratory frame. The runner carries the pole through the front door of a barn 10 m long. Just at the instant the head of the pole reaches the closed rear door, the front door can be closed, enclosing the pole within the 10-m barn for an instant. The rear door opens and the runner goes through. From the runner's point of view, however, the pole is 20 m long and the barn is only 5 m! Thus the pole can never be enclosed in the barn. Explain, quantitatively and by means of spacetime diagrams, the apparent paradox.