Homework 3

Due Friday, September 25th in class

Read Hecht's Ch. 2. (I also encourage you to read Ch. 1, which is a very nice historical summary. However, this is not required.)

Problem 1 There is a typo in the second paragraph of the Lab task I handed out to you last Wednesday.

(a) Find this typo and say what it is.

(b) Starting from the given claim about the intensity in this paragraph derive the equation at the end of the paragraph: $\Delta n = \frac{\Delta N \lambda_o}{L}$.

(c) Derive the equation at the end of the third paragraph of that hand out: $\Delta N = \frac{L(n_{\text{air}}-1)}{\lambda_o P_o}\Delta P$.

Rather than turn this in with the other homework, include it in the Analysis subsection of your report due Monday.

Problem 2 In class Monday, we will use the complex system of writing waves, $\psi(x,t) = A\cos(kx - \omega t + \phi) \rightarrow \tilde{\psi}(x,t) = Ae^{(i(kx-\omega t+\phi)}$, to show that the intensity of a superposition of two harmonic waves with the same frequency and different phases is dependent on the cosine of their phase differences. Consider two waves with different frequency, wavenumbers and phase, but the same speed: $\psi_1(x,t) = A_1\cos(k_1x - \omega_1 t)$ and $\psi_2(x,t) = A_2\cos(k_2x - \omega_2 t + \phi)$, with $\omega_1/k_1 = \omega_2/k_2 = c$. The intensity of a wave (or a quantity proportional to its intensity) is $I(x,t) = \frac{1}{\tau}\int_0^\tau \psi(x,t)^2 dt = \frac{1}{\tau}\int_0^\tau [\frac{1}{2}(\tilde{\psi}(x,t) + \tilde{\psi}(x,t)^*)]^2 dt$, where $\tilde{\psi}(x,t)^*$ is the complex conjugate of $\tilde{?}(x,t)$ and the τ interval must be an integer number of cycles for harmonic waves, that is, a multiple of the period.

(a) Use the complex notation formalism to show that the intensity for $\psi_1(x,t)$ alone is proportional to A_1^2 and the intensity of $\psi_2(x,t)$ alone is proportional to A_2^2.

(b) Use the complex notation formalism to show that the intensity of $\psi_1(x,t) + \psi_2(x,t)$ is equal to the sum of the intensities of the individual waves (i.e. there is no interference). Here the time-average interval τ should be an integer number of cycles of both $\psi_1(x,t)$ and $\psi_2(x,t)$.

(c) Finally use the complex notation to calculate the intensity of the superposition of two harmonic waves with the same frequency and wavenumber, different phases and amplitudes, and going in opposite directions: $\psi_1(x,t) = A_1\cos(k_1x - \omega_1 t) + A_2\cos(k_1x + \omega_1 t + \phi)$.

Problem 3 Examine the table and graphs of Figures 6 and 7 at the webpage:

`philiplaven.com/p20.html`,

which shows details of how electromagnetic waves travel in water. Note that the x-axes are in terms of the wavelength of these waves in vacuum. Answer the following questions for electromagnetic waves that have wavelengths of 1) 400nm, 2) 700nm, 3) 1mm, and 4) 10 m in vacuum.

(a) What is the speed of each of these waves in water?

(b) What is the (real) wavenumber of each of these waves in water?

(c) What is the angular frequency of each of these waves in water.

(d) Suppose each of these waves have intensity I_o when entering the water. How far does each travel in water before the intensity is reduced to $10^{-10} I_o$? (This reduction would render an almost overwhelmingly bright light imperceptible to the human eye). Can you make the connection between your answers and why water appears blue?

Problem 4 Find the first 5 terms of the Taylor expansions about $x = 0$ for each of the following functions:

(a) $\cos(kx)$ [k is a constant]

(b) $\sin(kx)$ [k is a constant]

(c) $e^{-\alpha x}$ [α is a constant]

(d) e^{ix} [i is the imaginary unit]

(e) Use your answers to show that the Euler relation $e^{ix} = \cos x + i \sin x$ holds for the first 5 terms of the Taylor expansion.

Problem 5 A useful aspect of Taylor expansions is using them to get approximate solutions to equations that are analytically difficult or impossible to solve. Take the example of a harmonic wave that begins to be absorbed on entering a medium at $x = 0$: $\psi(x) = Ae^{-\alpha x} \cos kx$ (we examine only the spatial dependence of the wave at $t = 0$), where $\alpha = 1\text{m}^{-1}$ and $k = 5\text{m}^{-1}$. Suppose you want to find the distance at which the amplitude of this wave is 90% of its amplitude at $x = 0$. In other words, for what x does $\psi(x) = Ae^{-\alpha x} \cos kx = 0.9A$? One can attempt to get an exact solution to this question by solving this equation analytically (feel free to try if you like), but if you want to avoid solving the problem this way, find an approximate solution by calculating the first 3 terms of the Taylor expansion of $\psi(x)$ about $x = 0$ and using this approximation in place of $\psi(x)$ to find the distance.