Homework 4

Due Friday, October 9th in class

Read Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 The relation between angular frequency ω, wavenumber k, and amplitude A for waves in deep water is often modeled by the relation $\omega^2 = gk(1 + (kA)^2)$, where g is the acceleration due to gravity. Using this model,

(a) What is the phase velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(b) What is the group velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(c) Is it possible to have a group of waves with the same phase and group velocities? If so, find a k and A for which this is true.

Problem 2 Refer back to Problem 3 of Homework 3 for the setup of this problem.

(a) Use Fig. 4 on the webpage `http://www.philiplaven.com/p20.html` to construct a sketch of $\omega(k)$ vs k for these wavelengths. (Again keep in mind that the x-axis is the wavelength of these waves in vacuum). Your sketch should have enough detail of the increase/decrease and curvature of $\omega(k)$ to see whether the group velocity is increasing or decreasing with k.

(b) Use the table of Fig. 6 to find the phase velocity of a 400nm (in vacuum) harmonic wave and a 425nm (in vacuum) harmonic wave.

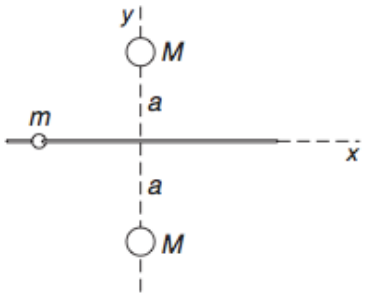
(c) What is the group velocity of the superposition of these two harmonic waves? Is it faster or slower than their phase velocities?

(d) Find the phase velocity of a 675 nm (in vacuum) harmonic wave and a 700 nm (in vacuum) harmonic wave.

(e) What is the group velocity of the superposition of the two harmonic waves in (d)? Is it faster or slower than their phase velocities?

(f) Which is faster, the group velocity of the first pair of harmonic waves or the second pair? Explain how one can see this from the sketch you drew in (a).

Problem 3 A bead of mass m slides without friction on a smooth rod along the x-axis. The rod is equidistant between two spheres of mass M. The spheres are located at $x = 0$, $y = \pm a$ as shown, and attract the bead gravitationally. Find the frequency of small oscillations of the bead about the origin.



Problem 4 A particle of mass m moves in one dimension along the positive x-axis. It is acted on by a constant force directed toward the origin with magnitude B, and an inverse-square law repulsive force with magnitude A/x^2. At what point x_o of the x-axis is the particle at a stable equilibrium? What is the frequency of small oscillations about the equilibrium point x_o?

Problem 5 A glass tube bent in the shape of a U contains water of density ρ. Initially the water is in equilibrium and fills each arm of the tube to a height h. Take the total length of the column of water to be ℓ. Then the whole column of water is displaced so that now the left arm contains water up to a height y above the equilibrium height and the water begins to oscillate. Assume that the tube has a circular cross section of radius r. What is the period of oscillations for this water column?