Homework 5

Due Friday, October 23rd in class

Read Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 (Air)

(a) What is the volume of one mole of air, at room temperature and 1 atm pressure?

(b) Estimate the number of air molecules in an average-sized room.

(c) Calculate the mass of a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).

Problem 2 (Exponential Atmosphere)

(a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for dP/dz, the variation of pressure with altitude, in terms of the density of air.

(b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. (The information needed to calculate m is given in the last problem.) Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P,$$

called the **barometric equation**.

(c) Assuming that the temperature of the atmosphere is independent of height (not a great assumption but not terrible either), solve the barometric equation to obtain the pressure as a function of height: $P(z) = P(0)e^{-mgz/kT}$. Show also that the density obeys a similar equation.

(d) Estimate the pressure, in atmospheres, at the following locations: Ogden, Utah (4,700 ft or 1,430 m above sea level); Leadville, Colorado (10,150 ft, 3,090 m); Mt. Whitney, California (14,500 ft, 4,420 m); Mt. Everest, Nepal! Tibet (29,000 ft, 8,840 m). (Assume that the pressure at sea level is 1 atm.)

Problem 3 Calculate the total thermal energy in a gram of lead at room temperature, assuming that none of the degrees of freedom are "frozen out" (this happens to be a good assumption in this case).

Problem 4 In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) What is the final volume of this air after compression?

(b) How much work is done in compressing the air?

(c) If the temperature of the air is initially 300 K, what is the temperature after compression?

Problem 5 In a Diesel engine, atmospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plugs.