Homework 6

Due Friday, October 30th in class

Reread Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 (The Atmosphere Again)
In Problem 2 of last week's homework you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however, the temperature of the bottommost 10-15 km of the atmosphere (called the tropo- sphere) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $|dT/dz|$ exceeds a certain critical value, convection will occur: Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must remain warmer than the surrounding air despite this adiabatic cooling.

(a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{dP} = \frac{2}{d+2}\frac{T}{P}$$

where d is the number of degrees of freedom of the gas.

(b) Assume that dT/dz is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. Use the result of Problem 2 (b) from last week to find a formula for dT/dz in this case. The result should be a constant, independent of temperature and pressure, which evaluates to approximately $-10°$ C/km. This fundamental meteorological quantity is known as the dry adiabatic lapse rate.

Problem 2 (Measuring Heat Capacities)
To measure the heat capacity of an object, all you usually have to do is put it in thermal contact with another object whose heat capacity you know. As an example, suppose that a chunk of metal is immersed in boiling water (100°C), then is quickly transferred into a Styrofoam cup containing 250 g of water at 20°C. After a minute or so, the temperature of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.

(a) How much heat is lost by the water?

(b) How much heat is gained by the metal?

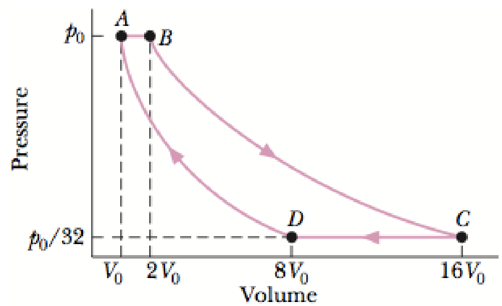
(c) What is the heat capacity of this chunk of metal?

(d) If the mass of the chunk of metal is 100 g, what is its specific heat capacity?

Problem 3 (Gaseous Engines)
An ideal gas (1 mol) is the working substance in an engine that operates on the cycle shown in the Figure. Processes BC and DA are reversible and adiabatic.

(a) Is the gas monatomic, diatomic, or polyatomic?

(b) What is the engine efficiency?



Problem 4 (Supercool)
Energy can be removed from water as heat at and even below the normal freezing point (0°C at atmospheric pressure) without causing the water to freeze; the water is then said to be supercooled. Suppose a 1 g water drop is supercooled until its temperature is that of the surrounding air, which is at −5.00°C. The drop then suddenly and irreversibly freezes, transferring energy to the air as heat. What is the entropy change for the drop? (Hint: Use a three-step reversible process as if the water were taken through the normal freezing point.) The specific heat of ice is 2220 J/kg · K and the Latent heat for ice is 333 J/g.

Problem 5 (Cooling a thermos)
An insulated Thermos contains 130 g of water at 80°C. You put in a 12 g ice cube at 0°C to form a system of *ice + original water*. (a) What is the equilibrium temperature of the system? What are the entropy changes of the water that was originally the ice cube (b) as it melts and (c) as it warms to the equilibrium temperature? (d) What is the entropy change of the original water as it cools to the equilibrium temperature? (e) What is the net entropy change of the *ice + original water* system as it reaches the equilibrium temperature?