Homework 8

Due Friday, November 20th in class

Read the excerpt from Griffiths' *Introduction to Quantum Mechanics* to solidify the discussion of probabilities that we had in class. I'll give you additional reading material as we get into quantum mechanics next week.

Problem 1 (Age distribution)
For the distribution of ages in Section 1.3.1 of the new handout:

(a) Compute $\langle j^2 \rangle$ and $\langle j \rangle^2$.

(b) Determine Δj for each j, and use Eq. 1.11 to compute the standard deviation.

(c) Use your results in (a) and (b) to check Equation 1.12.

Problem 2 (Rock and cliff)

(a) Find the standard deviation of the distribution of Example 1.1 of the handout.

(b) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

Problem 3 (Gaussian practice)
Consider the **gaussian** distribution

$$\rho(x) = Ae^{-\lambda(x-a)^2},$$

where A, a, and λ are positive real constants. (Recall how you memorized some aspects of integrating this distribution and learned how to compute others!)

(a) Use Eq. 1.16 of the handout to determine A.

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$ and σ.

(c) Sketch the graph of $\rho(x)$.

Problem 4 (Why electron microscopes?)
Transmission electron microscopes use high energy electrons to image biological samples, such as viruses, and for nanoscience research and development (for example, alloy particles and carbon nanotubes). What are the intrinsic limitations on the spatial resolution of such a microscope if the electrons are accelerated to energies between 40 and 100 keV? It is often true that the actual resolution is limited by the optics of the lens system and not by the fundamental limitation due to the de Broglie wavelength.

Problem 5 (Blackbody spectrum)
In class we derived the total energy in a cubical box of side length L and held at a temperature T:

$$E_{\text{tot}} = \int_0^\infty \frac{8\pi hV}{c^3} \frac{f^3}{e^{hf/kT} - 1} df,$$

where $V = L^3$ is the volume of the box. Find an expression for this total energy as an integral over wavelengths instead of frequency. [No need to rederive the result, just change variables in the equation above.]

Problem 6 (Wien's law)
Beginning with your result from Problem 5, define $u(\lambda, T)$ as the guts of integral you found in your answer divided by the volume V. This is known as the energy density (because it is divided by V) per unit wavelength. Find an equation that determines the maximum of $u(\lambda, T)$ in wavelength. This equation is not analytically solvable. Rewrite it in terms of the single variable $x = hc/(\lambda kT)$ and use the `NSolve` command in Mathematica to solve the equation numerically. Having determined a value for x figure out how the wavelength of the maximum of the spectrum depends on temperature and derive the value for α in Wien's law that I quoted in class.