Homework 9

Due Friday, December 4th in class

Read Chapter 1 and pp 24-40 of Chapter 2 from Griffiths' *Introduction to Quantum Mechanics*.

Problem 1 (Square well practice)
Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x and σ_p, for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

Problem 2 (Superpositions) A particle in the infinite square well has as its initial wave function an even mixture of the first two states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)].$$

(a) Normalize $\Psi(x, 0)$. (That is, find A. This is very easy, if you exploit the orthonormality of ψ_1 and ψ_2. Recall that, having normalized Ψ at $t = 0$, you can rest assured that it *stays* normalized—if you doubt this, check it explicitly after doing part (b).)

(b) Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$. Express the latter as a sinusoidal function of time, as in Example 2.1 in the text. To simplify the result, let $\omega \equiv \pi^2\hbar/2mL^2$, where L is the width of the well.

(c) Compute $\langle x \rangle$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (If your amplitude is greater than $L/2$, go directly to jail.)

(d) Compute $\langle p \rangle$. (As Peter Lorre would say, "Do it ze *kveek* vay, Johnny!"

(e) if you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of H. How does it compare with E_1 and E_2?

Problem 3 (Time dependence)
A particle of mass m in the infinite square well (of width L) starts out in the left half of the well, and is (at $t = 0$) equally likely to be found at any point in that region.

(a) What is its initial wave function, $\Psi(x, 0)$? (Assume it is real. Don't forget to normalize it.)

(b) Find $\Psi(x, t)$.

(c) What is the probability that a measurement of the energy would yield the value $\pi^2\hbar^2/2mL^2$?