Modern

Day 14

<u>Outline</u>

I Last time

II Wrap up group velocity

III What next?

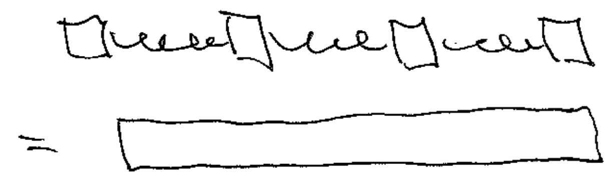
IV Pressure

with $v = \frac{KL^2}{M}$

- Realized that it was unreasonable to expect all waves (w/ differing k and ω) to have same speed due to unequal m's, $d \neq 0$ etc.
- This lead to the recognition that when you combine waves of different phase velocity you get new behavior

I

- Introduced the block and spring model for a material:



- Derived wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

II Going off of this, let's combine two harmonic waves with differing k and ω

$$\psi = A_1 e^{i(kx - \omega t)} + A_2 e^{i((k+\Delta k)x - (\omega + \Delta\omega)t)}$$

We find (as on your HW)

$$I \propto A_1^2 + A_2^2 + 2A_1A_2 \cos(\Delta k x - \Delta\omega t)$$

Intensity is a wave in its own right that moves with

$$v_g = \frac{\Delta\omega}{\Delta k} \xrightarrow{\text{limit}} \boxed{v_g = \frac{d\omega}{dk}}$$ "group velocity"

To distinguish we call

$v_p = \frac{\omega}{k}$ "phase velocity"

In fact, the group velocity is the more physical, hence important, quantity. When dispersion is linear

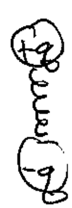
$\omega(k) = ck$

we get

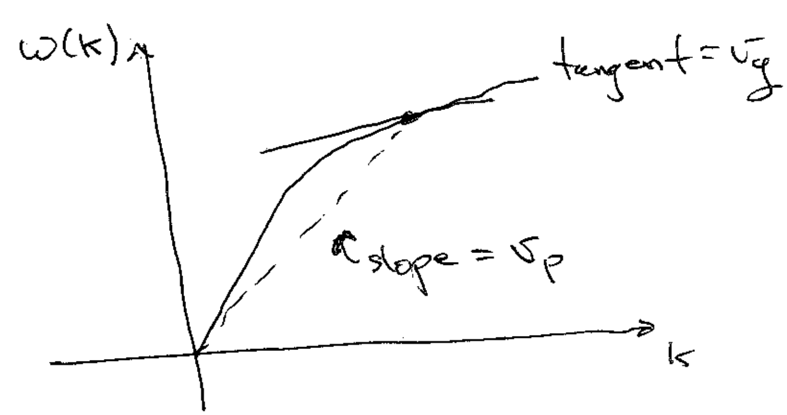
$\frac{d\omega}{dk} = c = \frac{\omega}{k}$ and $\boxed{v_g = v_p}$

(attenuation). But, they also create waves. How? Accelerating charges create E & M waves. How do the charges get set into motion? Again charges in a material must be in stable equilibrium ⟿ use spring model

Electromag. resonator



But, it's not true generally.



III We have noted that materials absorb E & M waves

How do you get it to vibrate? Heat, which was first thought of as a fluid, was later recognized to be the microscopic motion of atoms. You have seen heat at work, increase the temp. of an object and it

Starts to emit visible E&M waves.

In fact all objects at $T > 0°K$ produce E&M waves, they're just largely invisible.

Different materials emit different λ preferentially. This is due to particular details of how charge is arranged & can vibrate in materials. We see that to better understand how materials emit radiation we should study their bulk mechanical properties (e.g. pressure) and understand heat and temperature.

This is <u>thermodynamics</u>.

IV (1) <u>Pressure</u>: force per unit area: $P = F/A$

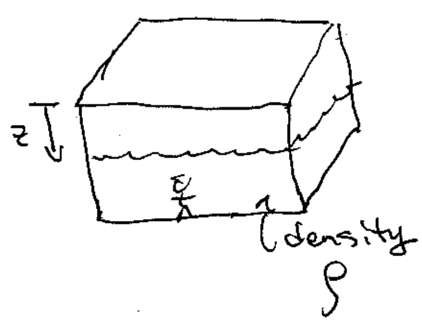
Units: $[P] = \frac{N}{m^2} \equiv 1\,Pa$ ← one Pascal

Pressure does not point (or if you like it points in all directions).

(2) How does pressure vary with depth?

It increases (z). Why?

Consider a slab of the fluid,



the pressure differential balances its weight. Force down on top surface

$$P(z) \cdot A$$

Force up on bottom surface:

$$P(z+dz) \cdot A$$

So, the net upward force is

$$P(z+dz) \cdot A - P(z)A = mg$$
$$= \rho A (dz) g$$

Then,

$$\frac{P(z+dz) - P(z)}{dz} = \rho g$$

or

$$\frac{dP}{dz} = \rho g.$$

For an incompressible fluid (water) ρ is independent of P. So,

$$P(z) = \rho g z + P(0)$$

↑ pressure at surface $z=0$.