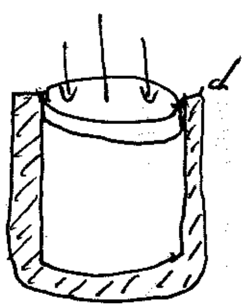
Modern
Day 19

Outline

I Last time

II Adiabatic Expansion of Ideal Gas

II Adiabatic means: no heat in or out. Question: How are P and V related during an adiabatic expansion?



1st Law: $đQ = \underbrace{dU}_{nC_v\,dT} + đW$, with $đQ = 0$ (adiabatic)

From the ideal gas law $PV = nRT$,

$$nR\,dT = d(PV) = P\,dV + V\,dP$$

I
- We introduced the molar heat capacity at constant volume and pressure:

$$C = \begin{cases} n\,C_v & \text{(const. vol.)} \\ n\,C_p & \text{(const. press.)} \end{cases}$$

- Discussed the difference between exact and inexact differentials
- $C_p = C_v + R$

$$\Rightarrow n\,dT = \frac{1}{R}(P\,dV + V\,dP)$$

Putting this into the 1st law gives

$$0 = C_v n\,dT + P\,dV$$

$$= \frac{C_v}{R}(P\,dV + V\,dP) + P\,dV$$

But $R = C_p - C_v$ and so,

$$0 = \frac{C_v}{C_p - C_v}(P\,dV + V\,dP) + P\,dV$$

Multiplying through by $C_p - C_v$ gives

$$0 = C_v(PdV + VdP) + (C_p - C_v)PdV$$

$$= C_v V dP + C_p P dV$$

Dividing through by V·P gives

$$0 = C_v \frac{dP}{P} + C_p \frac{dV}{V} \Rightarrow 0 = \frac{dP}{P} + \frac{C_p}{C_v}\frac{dV}{V}$$

Let $\gamma = C_p/C_v$. Integrating yields

$$\ln P + \gamma \ln V = \text{const}$$

$$\Rightarrow \boxed{PV^\gamma = \text{const.}}$$

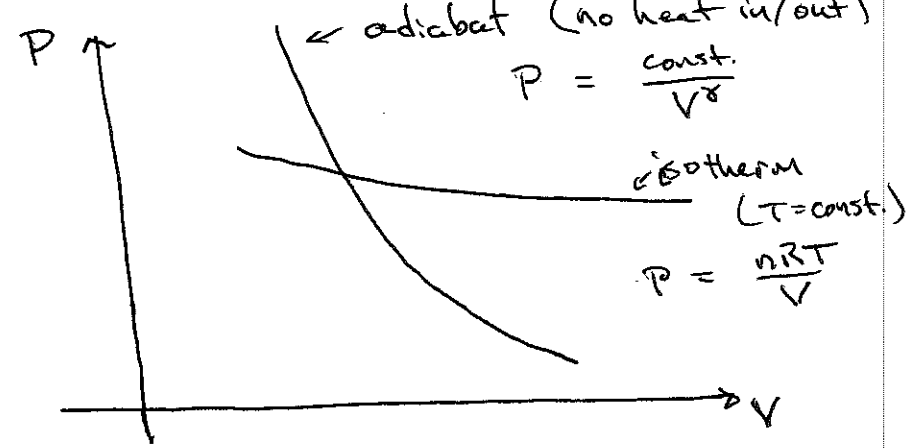
III Heat Engines

A device for converting heat into work. Typically some working substance carried around a cycle of heating and cooling.

<u>Efficiency</u> is defined in the natural way:

$$\epsilon = \frac{\Delta W}{\Delta Q_{in}} \quad \left(\begin{array}{c}\text{What you got out} \\ \text{divided by what} \\ \text{you had to pay}\end{array}\right)$$

This is often summarized on a PV diagram



Would <u>like</u>: $\epsilon = 1$ (100% conversion of heat to work.)

Unfortunately, every heat engine has an exhaust stroke in which heat ΔQ_{out} is expelled (wasted).