Modern
Day 2

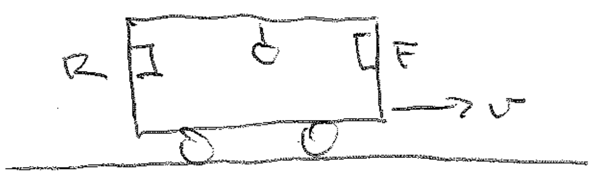
Sep 2nd, 2015 P1/4

Outline

I Last time

II More consequences

III Minkowski diagrams



(A) Observer on train:
R & F simultaneously

(B) Obs. on grnd:
R before F

Two events simult. to one (inertial) observer, may not be to another!

I. • Two postulate

(1) principle of relativity

(2) Universal speed of light

consistency requires Ein. vel. addition

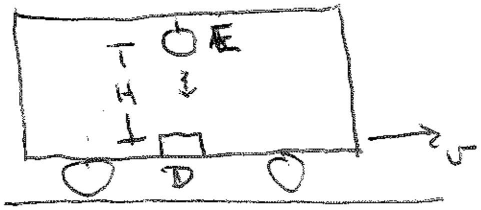
$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB} \cdot v_{BC}}{c^2}}$$

- Relativity of simultaneity

Point to add: <u>observation</u>: What you got after correcting for how long the message took to reach you. You could think of a custodian attached to each ref. frame

II (2) <u>Time Dilation</u>

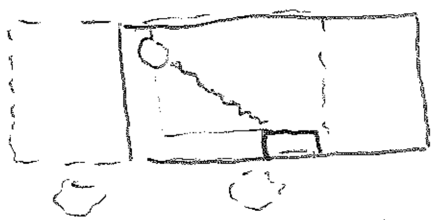
Another train experiment



How long btwn E and D?

(A) Observer on train: $\Delta t' = \frac{H}{c}$ $\frac{(\text{distance})}{(\text{speed})}$

(B) Observer on grnd:



$$\Delta t = \frac{\sqrt{H^2 + (v\Delta t)^2}}{c}$$

$$\Rightarrow c^2 \Delta t^2 = H^2 + v^2 \Delta t^2$$

or

$$\boxed{\Delta t' = \frac{1}{\gamma} \Delta t}$$

Conclusion: Moving clocks run slow — by a factor of γ.

Slow means they have fewer clicks, in other words you age more slowly when moving.

Example: $v = 3/4\,c$ then $\gamma = \frac{1}{\sqrt{1 - 9/16}} = \frac{1}{\sqrt{7/16}} = \frac{4}{\sqrt{7}}$

So,

$$\left(1 - \frac{v^2}{c^2}\right) \Delta t^2 = \frac{H^2}{c^2}$$

Now, let

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

(Note: $\gamma \geq 1$, with $\gamma = 1$ for $v = 0$ only)

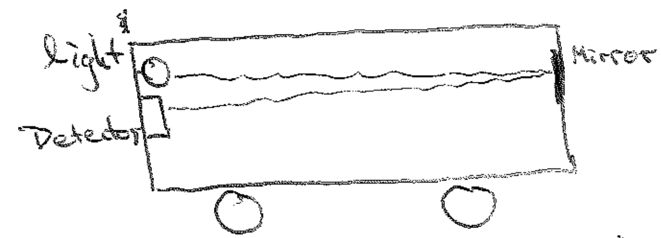
Then

$$\frac{1}{\gamma^2} \Delta t^2 = \Delta t'^2$$

So $\Delta t' = \frac{\sqrt{7}}{4}$ sec. for $\Delta t = 1$ sec.

$\approx .66$ sec.

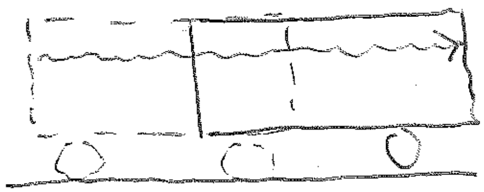
(3) <u>Lorentz Contraction</u>



Question: How long does the round trip take?

(A) on train; $\Delta t' = \frac{2L'}{c}$ leave open possibility of different L's

(B) On ground:



Δt_1 = interval after which light has reached mirror

$$\Delta t_1 = \frac{L + v\Delta t_1}{c}$$

$$\Rightarrow \Delta t_1\left(1 - \frac{v}{c}\right) = \frac{L}{c}$$

$$\Rightarrow \Delta t_1 = \frac{L/c}{1 - v/c}$$

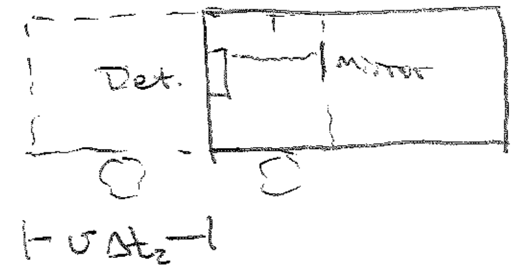
$$\Rightarrow \Delta t = \frac{L}{c}\left(\frac{1}{1 - v/c} + \frac{1}{1 + v/c}\right)$$

$$= \frac{L}{c}\left(\frac{1 + v/c + 1 - v/c}{1 - v^2/c^2}\right)$$

$$= \frac{2L}{c}\gamma^2$$

But, $\Delta t' = \frac{1}{\gamma}\Delta t$, so

$$\frac{2L'}{\cancel{c}} = \frac{1}{\gamma}\frac{2L}{\cancel{c}}\gamma^2$$



$$\Delta t_2 = \frac{L - v\Delta t_2}{c}$$

$$\Rightarrow \Delta t_2 = \frac{L/c}{1 + v/c}$$

Then

$$\Delta t = \Delta t_1 + \Delta t_2$$

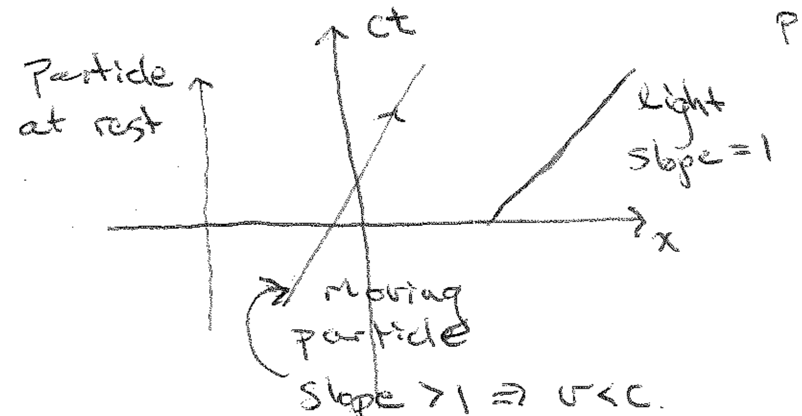
and

$$\boxed{L' = \gamma L}$$

<u>Conclusion</u>: $L' > L$, moving objects are shortened, by a factor of γ.

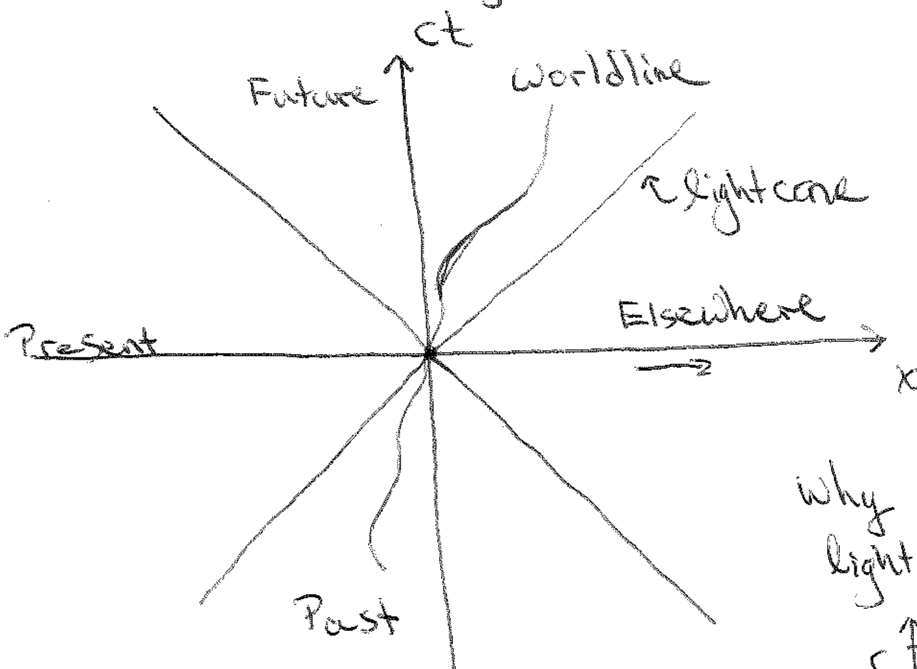
We'll derive (4) Einstein vel. addition one way on Friday, you'll do it another on the homework.

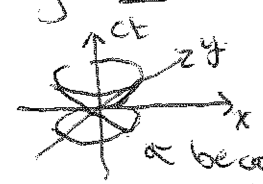
We've shown that when you move your notions of time and distance transform — and the ways in which they transform are related. This is a hint that it might be useful to consider them together and think of <u>Spacetime</u>.



Velocity (in units of c) is is now 1 over the slope.

II <u>Minkowski diagrams</u>: In relativity we plot x horizontally and ct vertically.





Separation of events