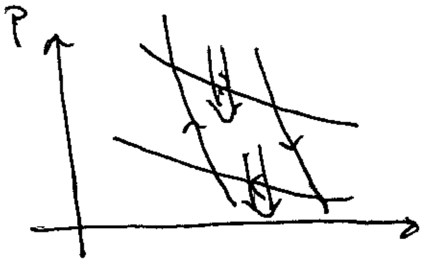
Modern Entropy: Day 22

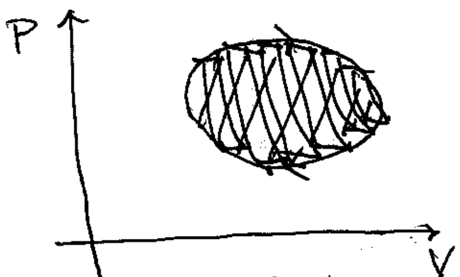
Today
0. More context for last time
I Last time
II Entropy as a state function and examples

0. Consider two blocks of aluminum, one 4 times bigger than the other. Heat the smaller one to 400 °C and the larger to 100°C. Which has had a more heat transferred to it?
A: Neither. But they have a different "quality".

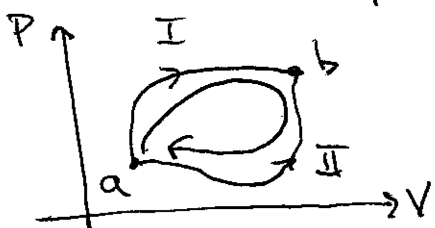
E.g. The larger one can heat up anything below 100°C while the smaller can heat anything below 400°C



Step 1: $\oint \frac{đQ}{T} = 0$



Step 2: $\oint \frac{đQ}{T} = 0$



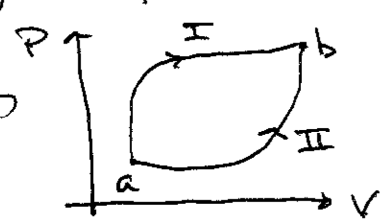
Step 3: $0 = \oint \frac{đQ}{T} = \int_{I} \frac{đQ}{T} - \int_{II} \frac{đQ}{T}$

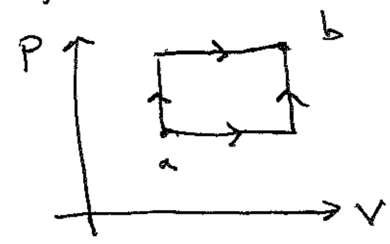
$\Rightarrow \int_{I} \frac{đQ}{T} = \int_{II} \frac{đQ}{T}$

I Last time we thought extensively about:

$$\int_a^b \frac{đQ}{T}$$

It is independent of the path taken from a to b:

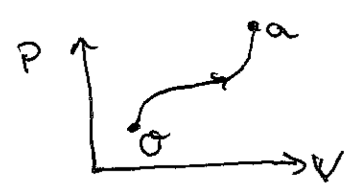




II Therefore we can define

$$S(a) = \int_{O}^{a} \frac{đQ}{T}$$

where O is an (arbitrary) reference point.



Called "the entropy of the system in state a" (relative to O).

Note: The difference in entropy between two states:

$\Delta S = S(b) - S(a) = \int_a^b \frac{đQ}{T}$

is independent of the reference point.

(A bit like altitude) Classical thermodynamics provides no natural reference point (like "sea level" for altitude) for entropy.

<u>Examples</u>: (1) 1 kg of ice melts, what's ΔS? (at 0°C in order for reversibility)

$\Delta S = \int \frac{đQ}{T} \overset{\text{melting at const. } T}{=} \frac{1}{T}\int đQ = \frac{1}{T} L_F m$

L_F = Specific latent heat of fusion
= energy required to change phase of matter / unit mass.

Then,

$\Delta S = \frac{1}{273\,K}(333 \times 10^3\ J/kg)\cdot 1\,kg$

$= \boxed{1.22 \times 10^3\ J/K}$

(2) Heat 1 kg of water from 0°C to 100°C, $\Delta S = ?$

$\Delta S = \int \frac{đQ}{T}, \quad đQ = mc\,dT$

$= \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln\left(\frac{T_f}{T_i}\right)$

$= (10^3\,gm)\left(4.19 \frac{J}{gm\,°K}\right) \ln\left(\frac{373}{273}\right)$

$= 1.31 \times 10^3\ J/K$

(3) Ideal gas: $(đQ = dU + đW)$

(i) <u>Constant T</u>.

$\Delta S = \int \frac{đQ}{T} = \frac{1}{T}\int đQ = \frac{1}{T}\int đW = \frac{1}{T}\int \frac{nRT}{V}dV = nR\ln\left(\frac{V_f}{V_i}\right)$

(ii) <u>Constant P</u>: $\Delta S = \int \frac{đQ}{T} = nC_p \int \frac{dT}{T}$

$\Delta S = nC_p \ln(T_f/T_i)$.

(iii) <u>Constant V</u>: $\Delta S = \int \frac{đQ}{T} = nC_v \int \frac{dT}{T}$

$\Delta S = nC_v \ln(T_f/T_i)$.

(iv) <u>Adiabatic</u>: $đQ = 0$ so

$\Delta S = 0$

Let's work through an <u>example of independence of path for ΔS</u>: (next time)