Modern day 3

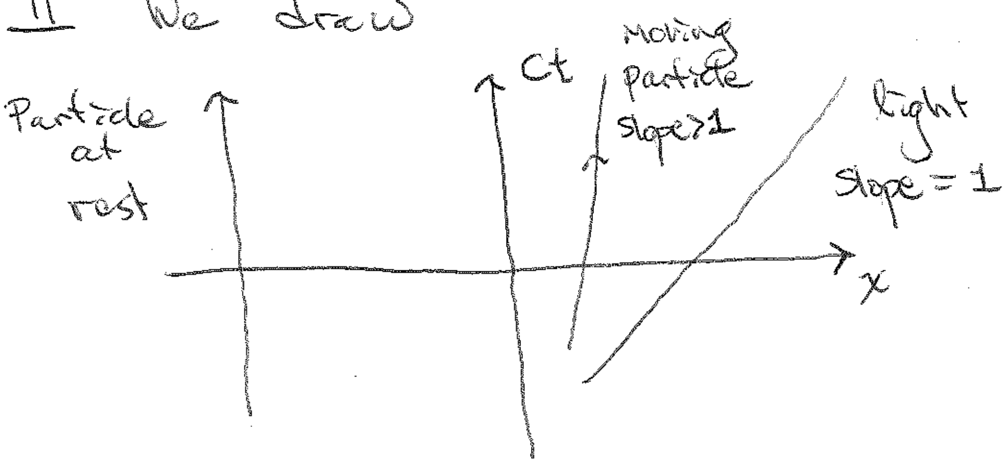
Outline

I Last time

II Minkowski diagrams

III Lorentz transformations

II We draw



$$\text{Slope} = \frac{c\Delta t}{\Delta x} = \frac{\text{rise}}{\text{run}}, \quad \text{but } \frac{\Delta x}{\Delta t} = v$$

$$= \frac{c}{v} \Rightarrow \boxed{v/c = \frac{1}{\text{slope}}}$$

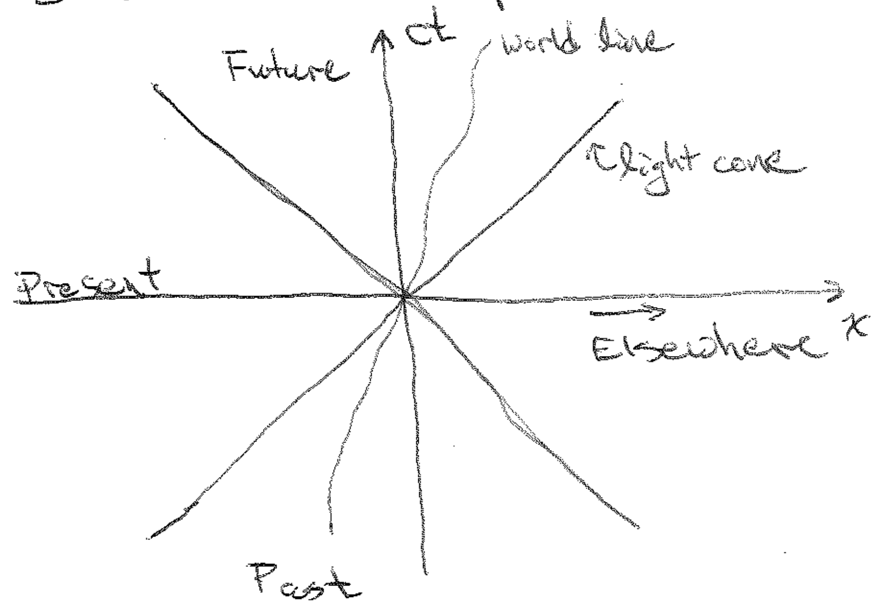
I • Derived two further consequences of Einstein's postulates:

time dilation: $\boxed{\Delta t' = \frac{1}{\gamma}\Delta t}$

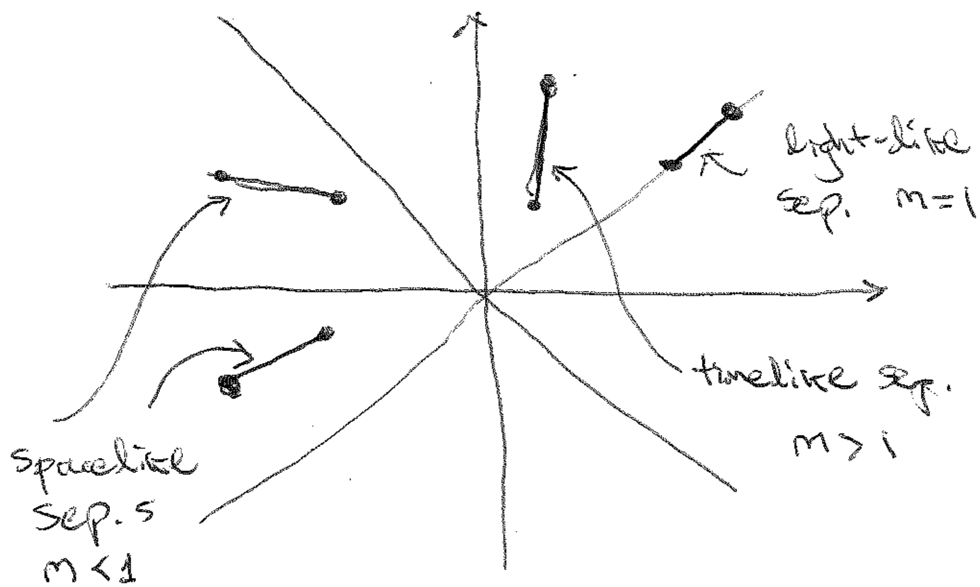
length contraction: $\boxed{\Delta L' = \gamma \Delta L}$

• Started to think about the unity of space and time

Velocity (in units of c) is 1 over the slope.

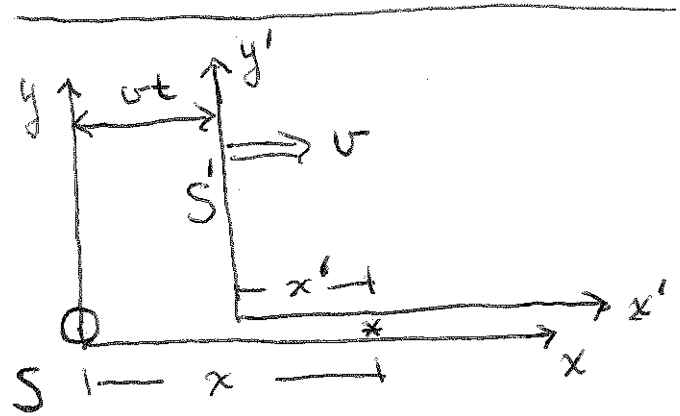


Separation of events



We seek a "dictionary" for translating language of S into the language of S'.

Galilean Transformation (pre-relativistic)



III Lorentz transformations

Two inertial Systems:

S ("rest" frame, "ground", "lab")

S' ("moving" frame, "train", "rocket")

Event: particular spacetime location (x, y, z), t (in S).

Question: What are the coords (x', y', z', t') of the SAME EVENT in S'?

Frame S' moves at speed v (const.) relative to S, along the x-axis.

At the instant the two origins coincide, set the two master clocks to zero.

After time t, the origin of S' has moved a distance vt.

Galilean trans.

$$\boxed{\begin{aligned} x' &= x - vt, & z' &= z, \\ y' &= y, & t' &= t. \end{aligned}}$$

Lorentz Transformations: (Relativistic)

Take perspective of S observer.

The Galilean "x'" failed to account for length contraction. Moving objects ~~appear~~ are shorter than in their rest frame, so

γ compensates.

$$x' = \gamma(x - vt)$$

$$y' = y \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$z' = z$$

$$t' = \gamma(t - v/c^2\, x)$$ ← Let's prove it.

$$\Rightarrow t' = \gamma\left(t + \frac{1}{v}\underbrace{\left(\frac{1-\gamma^2}{\gamma^2}\right)}_{= \frac{1}{\gamma^2} - 1 = \cancel{1} - \frac{v^2}{c^2} - \cancel{1}} x\right)$$

$$\Rightarrow t' = \gamma\left(t + \frac{1}{\cancel{v}}\left(-\frac{v^{\cancel{2}}}{c^2}\right)x\right)$$

$$\Rightarrow \boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)} \quad \checkmark$$

It's nicer to write

Inverse transformation

$$x = \gamma(x' + vt')$$

Then

$$x = \gamma[\gamma(x - vt) + vt']$$

$$\Rightarrow x = \gamma^2 x - \gamma^2 vt + \gamma v t'$$

$$\Rightarrow \gamma v t' = (1 - \gamma^2)x + \gamma^2 vt$$

$$\Rightarrow t' = \frac{(1-\gamma^2)}{\gamma v}x + \gamma t$$

$$\boxed{\begin{aligned} x' &= \gamma\left(x - \frac{v}{c}(ct)\right) \\ ct' &= \gamma\left((ct) - \frac{v}{c}x\right) \end{aligned}}$$

Ex: Non-Synchronization of clocks.

Say $t = 0$ in S; look at the S' clocks:

$$t' = \gamma(t - v/c^2\, x) = -\gamma \frac{v}{c^2}x$$

S' clocks