Outline

Modern

Day 7

0. Check in

I Example of using relativistic energy-momentum conservation.

II Spacetime diagrams again

0. Check in

How is speed of light measurement going?

How is Mathematica going?

How is Lyons reading going?

Announce Max's tutoring:

~~Ex~~ 7-9pm on Wed

Zecken: 1-3pm on Sun

Hal: M 5-6pm, Thurs. 5-6pm

Question: Relation btwn length contraction and time dilation and Lorentz transf.?

You measure length by finding the position of two events at the same moment. So,

$L = \Delta x$ with $\Delta t = 0$ then $L' = \Delta x' = \gamma \Delta x = \gamma L$

Serenity Prayer for Experiment:

"Grant me the Serenity to accept the things I cannot change,

The courage to change the things I can,

And the wisdom to know the difference"

greek mu.
↳ μ^- : muon, like a heavy electron

↳ ν_μ : muon neutrino, little neutral one
greek nu

$$m_\mu = 105.7 \frac{MeV}{c^2} = 1.884 \times 10^{-28}\ kg$$

$$m_{\nu_\mu} \sim \frac{1}{10}\ eV = 1.78 \times 10^{-37}\ kg$$ } 9 orders of mag.

↳ neglect and treat $m_{\nu_\mu} \approx 0$.

I Example: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ ← antiparticle

$$\Rightarrow \vec{P}_\nu = -\vec{P}_\mu \Rightarrow |\vec{P}_\nu| = |\vec{P}_\mu|$$

So, $m_\pi c^2 = E_\mu + |\vec{P}_\mu| c$

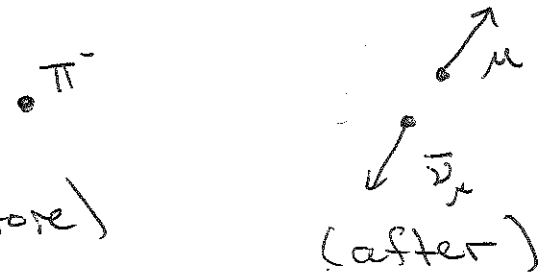
But, $P_\mu^2 c^2 = E_\mu^2 - m_\mu^2 c^4$

$$\Rightarrow |\vec{P}_\mu| c = \sqrt{E_\mu^2 - m_\mu^2 c^4}$$

Then

$$m_\pi c^2 = E_\mu + \sqrt{E_\mu^2 - m_\mu^2 c^4}$$

$$\Rightarrow (m_\pi c^2 - E_\mu)^2 = E_\mu^2 - m_\mu^2 c^4$$



What's the energy of the muon? Conservation of energy:

$$m_\pi c^2 = E_\mu + E_\nu \quad (E_\nu = |\vec{P}_\nu| c)$$

Cons. of momentum

$$0 = \vec{P}_\mu + \vec{P}_\nu$$

$$\Rightarrow m_\pi^2 c^4 - 2 m_\pi c^2 E_\mu + E_\mu^2 = E_\mu^2 - m_\mu^2 c^4$$

$$\Rightarrow \boxed{E_\mu = \frac{(m_\pi^2 + m_\mu^2)}{2 m_\pi} c^2}$$

Its momentum?

$$|\vec{P}_\mu| = \frac{1}{c}\left(\frac{(m_\pi^2 + m_\mu^2)^2}{4 m_\pi^2} c^4 - m_\mu^2 c^4 \right)^{1/2}$$

$$= c \left(\frac{m_\pi^4 + 2 m_\pi^2 m_\mu^2 + m_\mu^4 - 4 m_\pi^2 m_\mu^2}{4 m_\pi^2} \right)^{1/2}$$

$$\Rightarrow |\vec{p}_\mu| = c\left(\frac{(m_\pi^2 - m_\mu^2)^2}{4m_\pi^2}\right)^{1/2}$$

$$= \boxed{\frac{m_\pi^2 - m_\mu^2}{2m_\pi}c}$$

Suggestion 1: To get the E of a particle given its $\vec{p}$ (or vice versa), use the invariant

$$E^2 - |\vec{p}|^2c^2 = m^2c^4$$

Suggestion 2: If you know E and $\vec{p}$ of a particle and you want $\vec{v}$, use

$$\vec{v} = \frac{\vec{p}c^2}{E},$$

II Diagrams

Q: Can I draw another, moving observer's coordinates on the same diagram? Yes.

Q: What is the muon's speed? Well,

$$E = \gamma mc^2, \quad \vec{p} = \gamma m\vec{v}$$

So,

$$\frac{\vec{p}}{E} = \frac{\vec{v}}{c^2}$$

Then,

$$v_\mu = \frac{|\vec{p}|_\mu c^2}{E_\mu} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}c$$

$$ct' = \gamma(ct - \beta x), \quad \beta = \frac{v}{c}$$

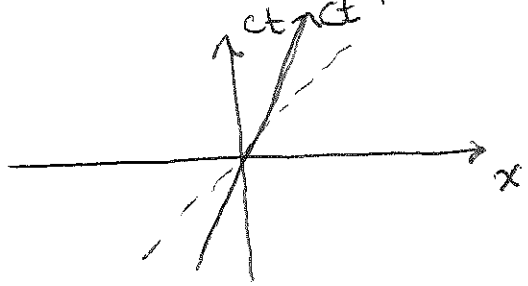
$$x' = \gamma(x - \beta ct)$$

Time axis for ct' is at $x' = 0$,

So,

$$0 = \gamma(x - \beta ct)$$

$$\Rightarrow ct = \frac{1}{\beta}x = \frac{c}{v}x \quad (t' \text{ axis})$$

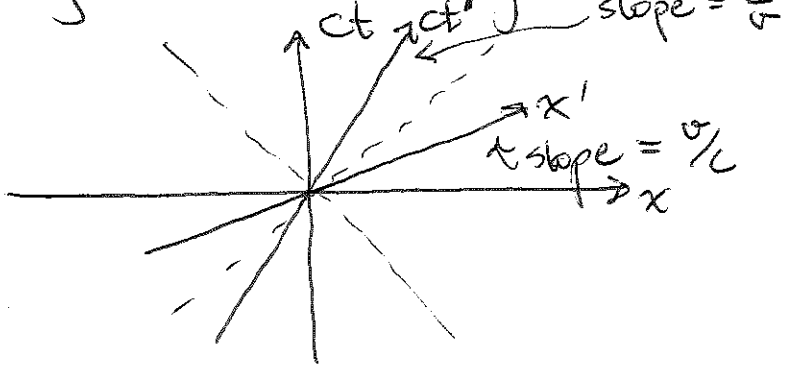


Similarly,

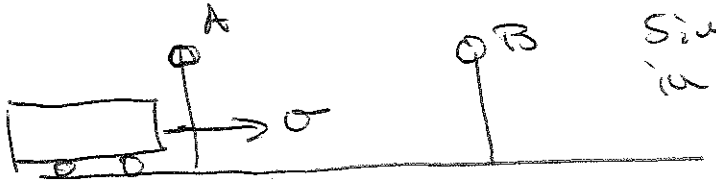
x'-axis: $ct' = 0 = \gamma(ct - \beta x)$

$\Rightarrow \quad ct = \beta x = \frac{v}{c} x$

Putting these all together

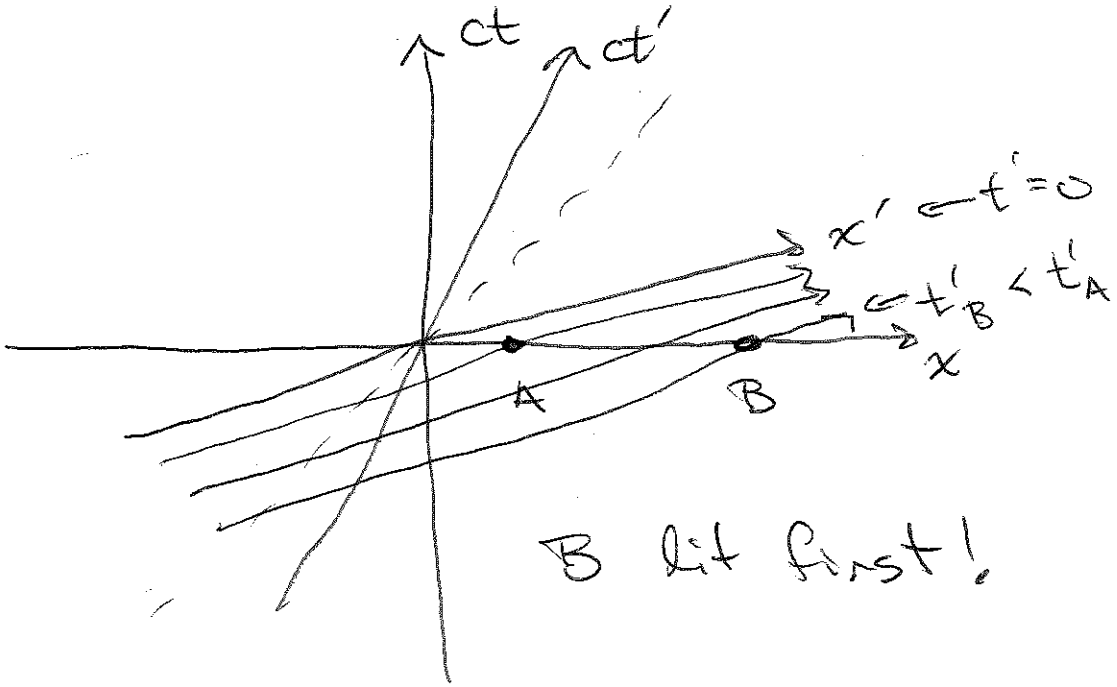


This gives us a graphical way to understand HW 1, problem 2:



A, B light simultaneously in ground frame

Does A or B light first according to observers on the train?



B lit first!