Modern Day 8

Outline

I Last time

II Michelson-Morley Experiment

III Wave motion

I. Conservation of E and $\vec{p}$ hold in relativity provided you use their relativistic definitions.

In doing calculations:

<u>Suggestion 0</u>: Work with E and $\vec{p}$, not $\vec{v}$.

<u>Suggestion 1</u>: Use

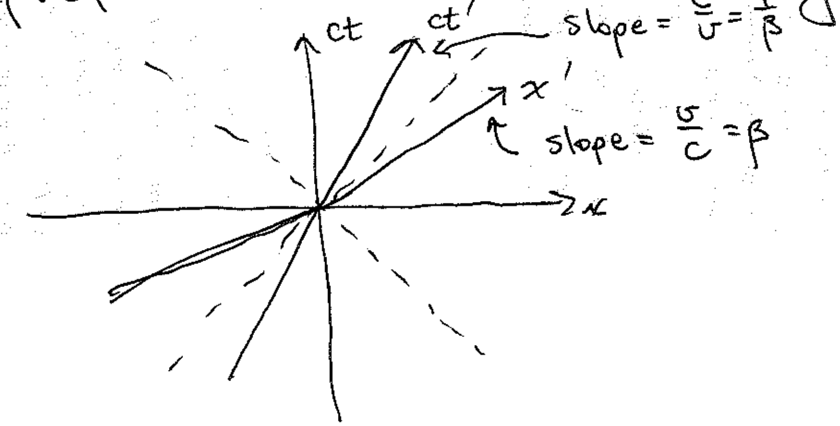
$$E^2 - p^2c^2 = m^2c^4$$

to get E given p and vice versa.

<u>Suggestion 2</u>: To get $\vec{v}$ give E and $\vec{p}$ use,

$$\vec{v} = \frac{\vec{p}c^2}{E}$$

Can depict a boosted frame using

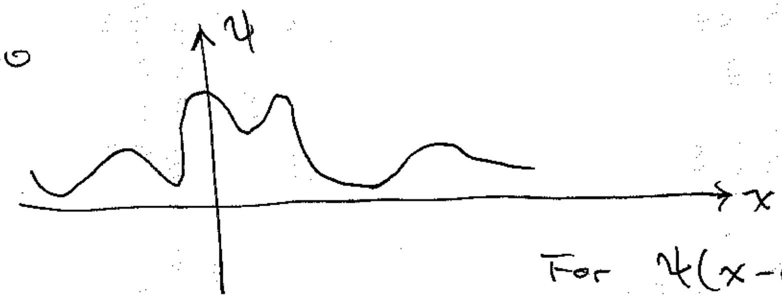


II Einstein's 2nd postulate highlights the special nature of light. But, so far we have treated light as an object, much like a particle. Part of the discovery of light's character came through its wave properties, to which we turn now.

The waves most familiar from are surroundings are the motion of energy and momentum through a medium; e.g. water, air, in wood (marimba), through rock (earthquake)...

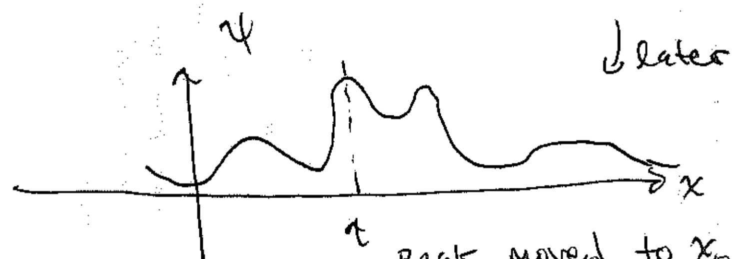
If light is an electromagnetic wave, what is the medium it is waving? This is the question that Michelson and Morley ~~asked~~.

$t=0$



For $\psi(x-ct)$

$t_0>0$ later time



peak moved to x_0

$$\psi(x+x_0, t_0) = \psi(x,0)$$

$$\Rightarrow \psi(x+x_0-ct_0) = \psi(x) \Rightarrow x - ct_0 = 0$$

didn't ask. But their experiment hinted at the answer.

III Wave motion

The wave equation (in 1D)

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} = c^2 \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

c is a constant $\rightarrow$ speed of wave.

Any differentiable function of the form $\psi(x-ct)$ or $\psi(x+ct)$ satisfies the wave equation.

We see, $\frac{x_0}{t_0} = c$ (positive) speed of wave.

$\psi(x+ct)$ soln.s are left-moving waves.

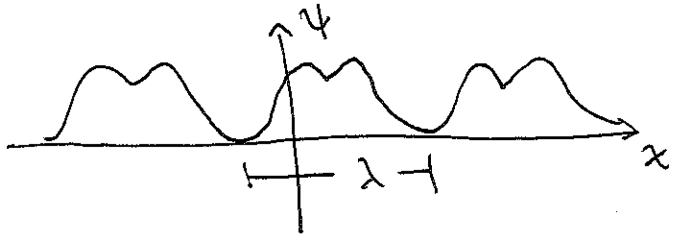
General property: <u>Superposition</u>

if $\psi_1(x,t)$ & $\psi_2(x,t)$ soln.s

the $\psi_1(x,t) + \psi_2(x,t)$ is too.

A (useful) subset of solns are the periodic waves:

(Eq 2) $\quad \psi(x+\lambda, t) = \psi(x,t)$



λ - wavelength, but then

$f = \frac{c}{\lambda}$ - call it frequency

$\frac{1}{f} = \tau = \frac{\lambda}{c}$ Period.

Eq. $\lambda \Rightarrow x + \lambda - ct = x - ct = x - c(t - \tau)$

$\Rightarrow \Psi(x, t - \tau) = \Psi(x, t)$ Periodic in time

Usual form

$A\cos(kx \pm kct + \phi) = A\cos(k(x+\lambda) \pm kct + \phi)$

$\Rightarrow k = \frac{2\pi}{\lambda}$ and $kc = \frac{2\pi}{T} = \omega$ "angular freq."

$= A\cos(kx \pm \omega t + \phi)$

Harmonic waves are useful:

1. sin/cos are easy to manipulate
2. Any periodic wave = sum (possibly ∞) of harmonic waves (Fourier Series)
3. Any wave over finite range of x & t can be written as sum of harmonic waves

Further subset: harmonic ~~solns~~ waves (sinusoidal soln.s)

$\psi(x,t) = A\sin(x \pm ct),\ B\cos(x \pm ct)$

Of course, sin and cos are just offsets of one another, we write

$$\underset{\text{Amplitude}}{A}\cos(\underset{\text{wave \#}}{k}[x \pm ct] + \underset{\text{phase or phase shift}}{\phi})$$

Even easier to use than sin's and cos's is

$$\boxed{e^{\pm i\theta} = \cos\theta \pm i\sin\theta}$$ Euler's relation

$\Rightarrow \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

This will be our theme next time.