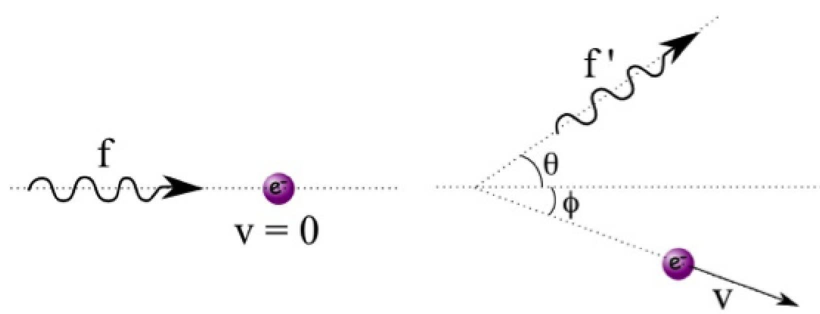
Homework 10

Due Friday, November 30th at 5pm

Read Chapter 1 and pp 24-40 of Chapter 2 from Griffiths' *Introduction to Quantum Mechanics*.

Problem 1 (Compton Scattering Setup)
Consider a collision between a photon traveling along the x-axis and an electron initially at rest. After the collision, the electron travels with velocity v at an angle ϕ with respect to the x-axis and the photon travels at an angle θ with respect to the x-axis. The photon frequency may change after the collision from f to f', but, though the collision must be treated relativistically, the electron does not gain mass (it has no internal structure that can store localized energy). Express your answers to the questions below in terms of f, f' (or the corresponding wavelengths λ and λ'), the mass of the electron m, the speed of the electron v after the collision, the angles θ and ϕ, the relativistic γ factor, and fundamental constants.



(a) What is the energy of the photon before the collision? What is the energy of the photon after the collision?

(b) What is the energy of the electron before the collision? What is the energy of the electron after the collision?

(c) Write down an equality based on the conservation of energy in the collision.

(d) What is the momentum of the photon before the collision? What are the two components of the photon momentum after the collision?

(e) What is the momentum of the electron before the collision? What are the two components of the electron's momentum after the collision?

(f) Write down two equalities based on the conservation of momentum in each direction in the collision.

Problem 2 (Compton Scattering)
Starting from the three conservation equations in **Problem 1** (c) and (f) derive the following relation:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Here's a strategy: First, use the momentum equations and the relation $\cos^2\phi + \sin^2\phi = 1$ to eliminate ϕ. Next use the energy equation and the relativistic relation $E^2 = p^2c^2 + m^2c^4$ to find an expression for the square of the momentum of the electron that does not depend on v (or γ). Finally, use this expression to eliminate v and γ from the momentum relation.

Problem 3 (Characterizing a wave function)
Consider the wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t},$$

where A, λ, and ω are positive real constants.

(a) Normalize Ψ.

(b) Determine the expectation values of x and x^2.

(c) Find the standard deviation of x. Sketch the graph of $|\Psi|^2$, as a function of x, and mark the points $(\langle x\rangle + \sigma)$ and $(\langle x\rangle - \sigma)$, to illustrate the sense in which σ represents the "spread" in x. What is the probability that the particle would be found outside this range?

Problem 4 (Square well practice)
Calculate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$, σ_x and σ_p, for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

Problem 5 (Superpositions) A particle in the infinite square well has as its initial wave function an even mixture of the first two states:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

(a) Normalize $\Psi(x,0)$. (That is, find A. This is very easy, if you exploit the orthonormality of ψ_1 and ψ_2. Recall that, having normalized Ψ at $t=0$, you can rest assured that it *stays* normalized—if you doubt this, check it explicitly after doing part (b).)

(b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time, as in Example 2.1 in the text. To simplify the result, let $\omega \equiv \pi^2\hbar/2mL^2$, where L is the width of the well.

(c) Compute $\langle x\rangle$. Notice that it oscillates in time. What is the angular frequency of the oscillation? What is the amplitude of the oscillation? (If your amplitude is greater than $L/2$, go directly to jail.)

(d) Compute $\langle p\rangle$. (As Peter Lorre would say, "Do it ze *kveek* vay, Johnny!")

(e) if you measured the energy of this particle, what values might you get, and what is the probability of getting each of them? Find the expectation value of H. How does it compare with E_1 and E_2?