Homework 11

Due Friday, December 7th at 5pm

Read Chapter 1 and pp 24-40 of Chapter 2 from Griffiths' *Introduction to Quantum Mechanics*.

Problem 1 (Massive particle phase and group velocities)
Now that we have identified massive objects with waves, it makes sense to find the dispersion relation $\omega(k)$ for these objects and the associated wave velocities. Using the relativistic energy-momentum relation $E^2 = p^2c^2 + m^2c^4$ and the quantum relations $E = h\nu = \hbar\omega$ and $p = h\lambda = \hbar k$:

(a) Find the phase velocity of a relativistic object in terms of its mass, momentum, and c.

(b) Find the group velocity of a relativistic object in terms of its mass, momentum, and c.

(c) What property would an object have to have for its phase and group velocity to be the same? What would its velocity be?

(d) Comparing the phase and group velocities when different, how do each compare to c? Based on your answer, should one consider the phase or the group velocity to be the appropriate velocity assigned to the object?

Problem 2 (Time-dependent wave function 1)
Consider the initial wave function

$$\Psi(x, 0) = Ax, \quad \text{for} \quad 0 < x < a,$$

in the infinite square well. Here A is a positive real constant.

(a) Normalize $\Psi(x, 0)$.

(b) Use Fourier's trick to find the constants c_n for this initial state.

(c) Write out the full, time-dependent solution to Schrödinger's equation $\Psi(x, t)$.

Problem 3 (Time-dependent wave function 2)
Consider the initial wave function

$$\Psi(x, 0) = Ax(a - x), \quad \text{for} \quad 0 < x < a,$$

in the infinite square well. Here A is a positive real constant.

(a) Normalize $\Psi(x, 0)$.

(b) Use Fourier's trick to find the constants c_n for this initial state.

(c) Write out the full, time-dependent solution to Schrödinger's equation $\Psi(x, t)$.