Homework 4

Due Friday, October 12th at 5pm

Read Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 The relation between angular frequency ω, wavenumber k, and amplitude A for waves in deep water is often modeled by the relation $\omega^2 = gk(1 + (kA)^2)$, where g is the acceleration due to gravity. Using this model,

(a) What is the phase velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(b) What is the group velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(c) Is it possible to have a group of waves with the same phase and group velocities? If so, find a k and A for which this is true.

Problem 2 Examine the table and graphs of Figures 6 and 7 at the webpage:

`philiplaven.com/p20.html`,

which shows details of how electromagnetic waves travel in water. Note that the x-axes are in terms of the wavelength of these waves in vacuum. Answer the following questions for electromagnetic waves that have wavelengths of (i) 400nm, (ii) 700nm, (iii) 1mm, and (iv) 10 m in vacuum:

(a) What is the speed of each of these waves in water?

(b) What is the (real) wavenumber of each of these waves in water?

(c) What is the angular frequency of each of these waves in water.

(d) Suppose each of these waves have intensity I_o when entering the water. How far does each travel in water before the intensity is reduced to $10^{-10}I_o$? (This reduction would render an almost overwhelmingly bright light imperceptible to the human eye). Make a connection between your answers and why water appears blue.

Please explain how you answer each type of question in words, but collect all your answers in a single table. This will be helpful for you to see comparisons between your results.

Problem 3 This problem has the same setup as the previous problem.

(a) Use Fig. 4 on the webpage `philiplaven.com/p20.html`, to construct a sketch of $\omega(k)$ vs k for these wavelengths. (Again keep in mind that the x-axis is the wavelength of these waves in vacuum). Your sketch should have enough detail of the increase/decrease and curvature of $\omega(k)$ to see whether the group velocity is increasing or decreasing with k.

(b) Use the table of Fig. 6 to find the phase velocity of a 400nm (in vacuum) harmonic wave and a 425nm (in vacuum) harmonic wave.

(c) What is the group velocity of the superposition of these two harmonic waves? Is it faster or slower than their phase velocities?

(d) Find the phase velocity of a 675 nm (in vacuum) harmonic wave and a 700 nm (in vacuum) harmonic wave.

(e) What is the group velocity of the superposition of the two harmonic waves in (d)? Is it faster or slower than their phase velocities?

(f) Which is faster, the group velocity of the first pair of harmonic waves or the second pair? Explain how one can see this from the sketch you drew in (a).