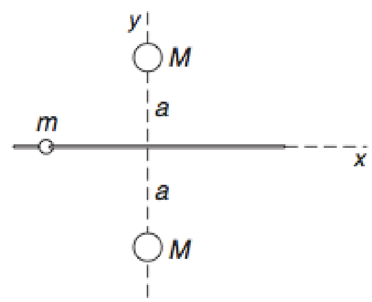
Homework 5

Due Friday, October 19th at 5pm

Read Chapter 1 of Schroeder's book *Thermal Physics*.

Problem 1 A bead of mass m slides without friction on a smooth rod along the x-axis. The rod is equidistant between two spheres of mass M. The spheres are located at $x = 0$, $y = \pm a$ as shown, and attract the bead gravitationally. Find the frequency of small oscillations of the bead about the origin.



Problem 2 A glass tube bent in the shape of a U contains water of density ρ. Initially the water is in equilibrium and fills each arm of the tube to a height h. Take the total length of the column of water to be ℓ. Then the whole column of water is displaced so that now the left arm contains water up to a height y above the equilibrium height and the water begins to oscillate. Assume that the tube has a circular cross section of radius r. What is the period of oscillations for this water column?

Problem 3 (Air)

(a) What is the volume of one mole of air, at room temperature and 1 atm pressure?

(b) Estimate the number of air molecules in an average-sized room.

(c) Calculate the mass of a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).

Problem 4 (Exponential Atmosphere)

(a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for dP/dz, the variation of pressure with altitude, in terms of the density of air.

(b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. (The information needed to calculate m is given in the last problem.) Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P,$$

called the **barometric equation**.

(c) Assuming that the temperature of the atmosphere is independent of height (not a great assumption but not terrible either), solve the barometric equation to obtain the pressure as a function of height: $P(z) = P(0)e^{-mgz/kT}$. Show also that the density obeys a similar equation.

(d) Estimate the pressure, in atmospheres, at the following locations: Ogden, Utah (4,700 ft or 1,430 m above sea level); Leadville, Colorado (10,150 ft, 3,090 m); Mt. Whitney, California (14,500 ft, 4,420 m); Mt. Everest, Nepal! Tibet (29,000 ft, 8,840 m). (Assume that the pressure at sea level is 1 atm.)

Problem 5 Calculate the total thermal energy in a gram of lead at room temperature, assuming that none of the degrees of freedom are "frozen out" (this happens to be a good assumption in this case).