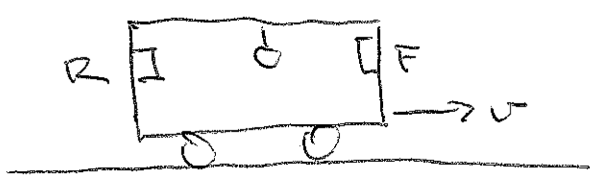
Modern
Day 2

Outline

I Last time

II More consequences

III Minkowski diagrams



(A) Observer on train:
R & F simultaneously

(B) Obs. on grnd:
R before F

Two events simult. to one (inertial) observer, may not be to another!

I. • Two postulate

(1) principle of relativity

(2) Universal speed of light

consistency requires Ein. vel. addition

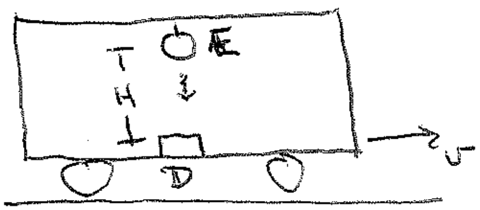
$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB} \cdot v_{BC}}{c^2}}$$

• Relativity of simulteneity

Point to add: observation:
What you got after correcting for how long the message took to reach you. You could think of a custodian attached to each ref. frame

II (2) Time Dilation

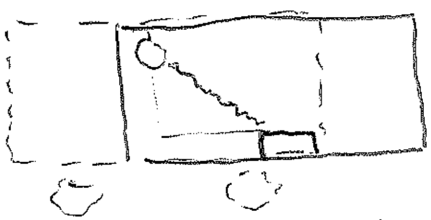
Another train experiment



How long btwn E and D?

(A) Observer on train: $\Delta t' = \frac{H}{c}$ $\frac{(\text{distance})}{(\text{speed})}$

(B) Observer on grnd:



$$\Delta t = \frac{\sqrt{H^2 + (v\Delta t)^2}}{c}$$

$$\Rightarrow c^2 \Delta t^2 = H^2 + v^2 \Delta t^2$$

So,

$$\left(1 - \frac{v^2}{c^2}\right) \Delta t^2 = \frac{H^2}{c^2}$$

Now, let

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

(Note: $\gamma \geq 1$, with $\gamma = 1$ for $v = 0$ only)

Then

$$\frac{1}{\gamma^2} \Delta t^2 = \Delta t'^2$$

or

$$\boxed{\Delta t' = \frac{1}{\gamma} \Delta t}$$

Conclusion: Moving clocks run slow — by a factor of γ.

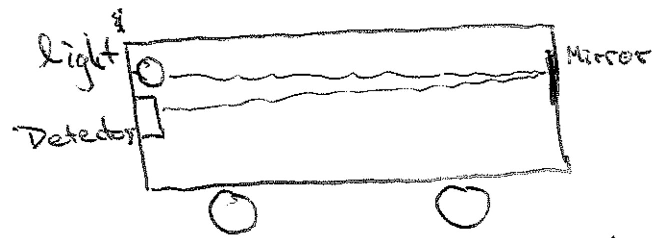
Slow means they have fewer clicks, in other words you age more slowly when moving.

Example: $v = 3/4 c$ then $\gamma = \frac{1}{\sqrt{1 - 9/16}} = \frac{1}{\sqrt{7/16}} = \frac{4}{\sqrt{7}}$

So $\Delta t' = \frac{\sqrt{7}}{4}$ sec. for $\Delta t = 1$ sec.

$\approx .66$ sec.

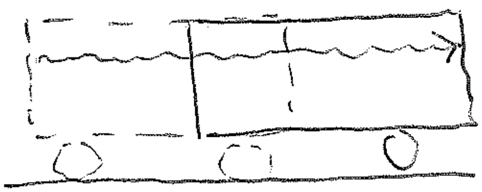
(3) Lorentz Contraction



Question: How long does the round trip take?

(A) on train; $\Delta t' = \frac{2L'}{c}$ ← leave open possibility of different L's

(B) On ground:



Δt_1 = interval after which light has reached mirror

$$\Delta t_1 = \frac{L + v\Delta t_1}{c}$$

$$\Rightarrow \Delta t_1\left(1 - \frac{v}{c}\right) = \frac{L}{c}$$

$$\Rightarrow \Delta t_1 = \frac{L/c}{1 - v/c}$$

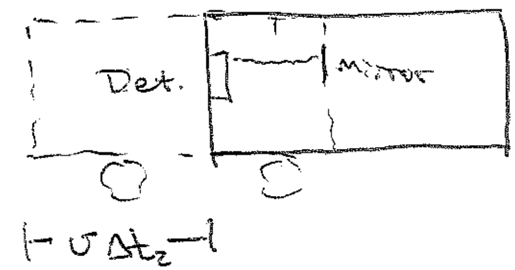
$$\Rightarrow \Delta t = \frac{L}{c}\left(\frac{1}{1 - v/c} + \frac{1}{1 + v/c}\right)$$

$$= \frac{L}{c}\left(\frac{1 + v/c + 1 - v/c}{1 - v^2/c^2}\right)$$

$$= \frac{2L}{c}\gamma^2$$

But, $\Delta t' = \frac{1}{\gamma}\Delta t$, so

$$\frac{2L'}{\cancel{c}} = \frac{1}{\cancel{\gamma}}\frac{2L}{\cancel{c}}\gamma^{\cancel{2}}$$



$$\Delta t_2 = \frac{L - v\Delta t_2}{c}$$

$$\Rightarrow \Delta t_2 = \frac{L/c}{1 + v/c}$$

Then

$$\Delta t = \Delta t_1 + \Delta t_2$$

and

$$\boxed{L' = \gamma L}$$

<u>Conclusion</u>: $L' > L$, moving objects are shortened, by a factor of γ.

We'll derive (4) Einstein vel. addition one way on Friday, you'll do it another on the homework.