Modern day 3

Outline

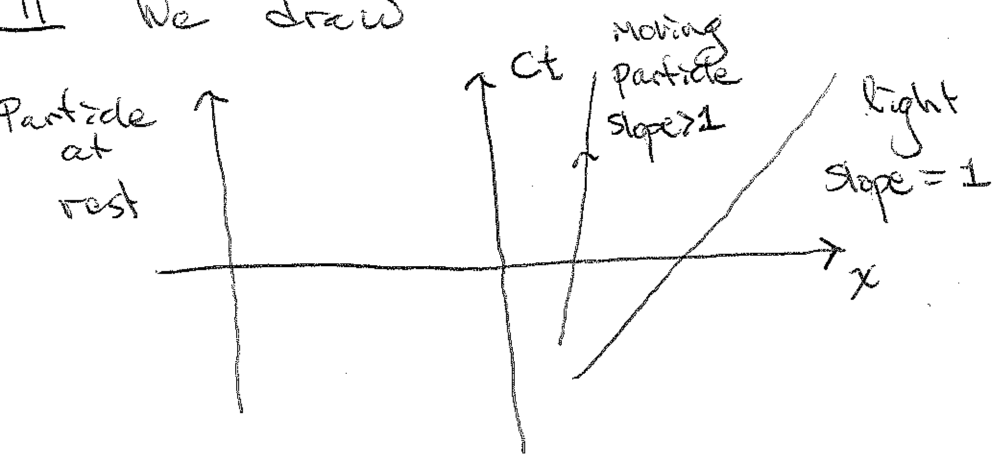
I Last time

II Minkowski diagrams

III Lorentz transformations

I • We've shown that when you move your notions of time and length change — and the ways in which they transform are related. This is a hint that it might be useful to consider them together and think of <u>Spacetime</u>.

II We draw



$$\text{Slope} = \frac{c\Delta t}{\Delta x} = \frac{\text{rise}}{\text{run}}, \text{ but } \frac{\Delta x}{\Delta t} = v$$

$$= \frac{c}{v} \Rightarrow \boxed{v/c = \frac{1}{\text{slope}}}$$

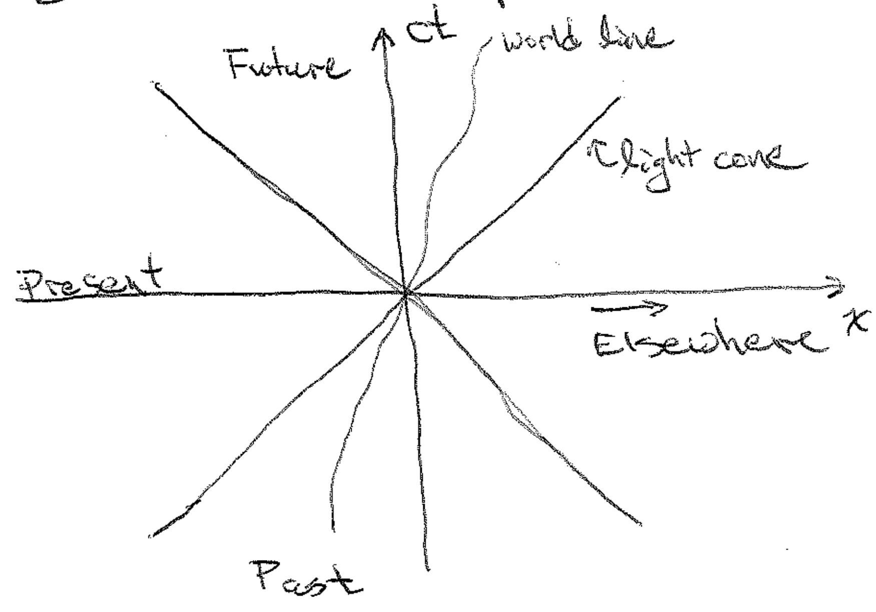
• Derived two further consequences of Einstein's postulates:

<u>time dilation</u>: $\boxed{\Delta t' = \frac{1}{\gamma}\Delta t}$

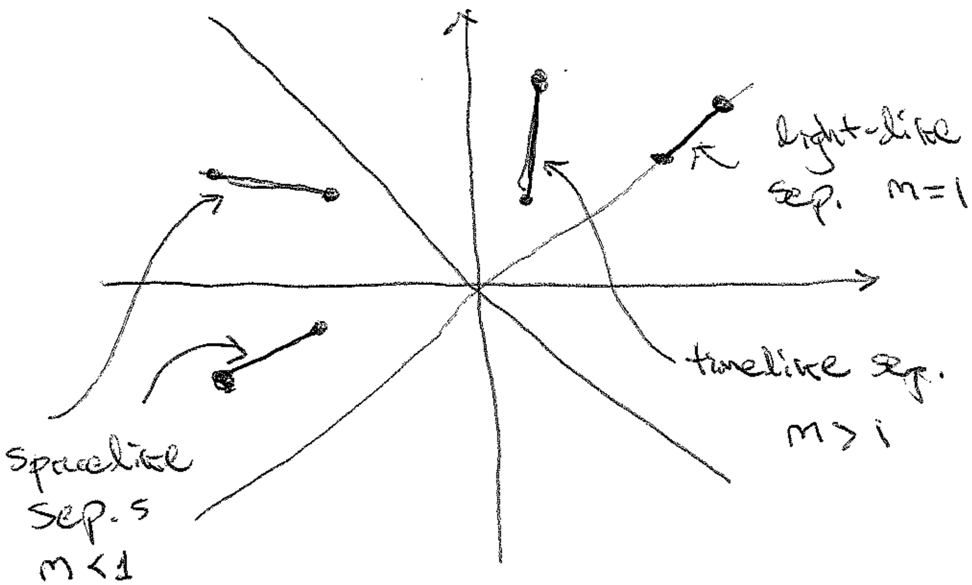
<u>length contraction</u>: $\boxed{\Delta L' = \gamma \Delta L}$

• Started to think about the unity of space and time

Velocity (in units of c) is 1 over the slope.

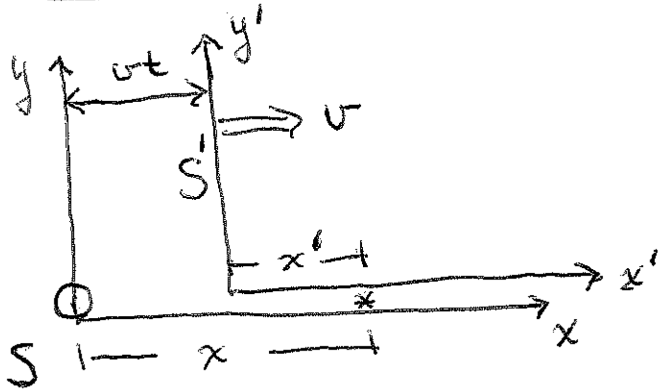


Separation of events



We seek a "dictionary" for translating language of S into the language of S'.

Galilean Transformation (pre-relativistic)



III Lorentz transformations

Two inertial Systems:

S ("rest" frame, "ground", "lab")

S' ("moving" frame, "train", "rocket")

Event: particular spacetime location (x, y, z), t (in S).

Question: What are the coord.s (x', y', z', t') of the SAME EVENT in S'?

Frame S' moves at speed v (const.) relative to S, along the x-axis.

At the instant the two origins coincide, set the two master clocks to zero.

After time t, the origin of S' has moved a distance vt.

Galilean trans.

$$\boxed{\begin{array}{ll} x' = x - vt, & z' = z, \\ y' = y, & t' = t. \end{array}}$$

Lorentz Transformations: (Relativistic)

Take perspective of S observer.

The Galilean "x'" failed to account for length contraction. Moving objects ~~appear~~ are shorter than in their rest frame, so ← compensates.

$$x' = \gamma\,(x - vt) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\,(t - v/c^2\, x)$$ ← Let's prove it.

Inverse transformation

$$x = \gamma\,(x' + vt')$$

Then

$$x = \gamma\left[\gamma(x - vt) + vt'\right]$$

$$\Rightarrow x = \gamma^2 x - \gamma^2 v t + \gamma v t'$$

$$\Rightarrow \gamma v t' = (1 - \gamma^2)x + \gamma^2 v t$$

$$\Rightarrow t' = \frac{(1-\gamma^2)}{\gamma v}x + \gamma t$$

$$\Rightarrow t' = \gamma\left(t + \frac{1}{v}\underbrace{\left(\frac{1-\gamma^2}{\gamma^2}\right)}_{= \frac{1}{\gamma^2} - 1 = \cancel{1} - \frac{v^2}{c^2} - \cancel{1}}x\right)$$

$$\Rightarrow t' = \gamma\left(t + \frac{1}{\cancel{v}}\left(-\frac{v^{\cancel{2}}}{c^2}\right)x\right)$$

$$\Rightarrow \boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)} \quad \checkmark$$

It's nicer to write

$$\boxed{\begin{aligned} x' &= \gamma\left(x - \frac{v}{c}(ct)\right) \\ ct' &= \gamma\left((ct) - \frac{v}{c}x\right) \end{aligned}}$$

Ex: Non-Synchronization of clocks.

Say $t=0$ in S; look at the S' clocks:

$$t' = \gamma\,(t - v/c^2\, x) = -\gamma\frac{v}{c^2}x$$

↳ S

S' clocks