

Outline

- I Last time
- II Energy & Momentum in Relativity
- III Conservation of relativistic energy/mom.
- IV Massless particles

Proper time: particle's wrist watch time, call it τ .

$$d\tau = \gamma dt$$

Proper velocity

$$\eta = \frac{\Delta x}{\Delta \tau} = \gamma \frac{\Delta x}{\Delta t} = \gamma v$$

or more generally

$$\eta^\mu = \frac{dx^\mu}{d\tau} = \gamma (c, v_x, v_y, v_z).$$

Modern

Day 5

I

Lorentz transformations

P1/3

$$\begin{aligned} (x^0)' &= \gamma (x^0 - \beta x^1) \\ (x^1)' &= \gamma (x^1 - \beta x^0) \\ (x^2)' &= x^2 \\ (x^3)' &= x^3 \end{aligned}$$

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$$

A four-vector a^μ is any collection of 4 #'s that transforms like x^μ does.

Note that

$$\gamma = \frac{1}{\sqrt{1 - \frac{1}{\gamma^2} \frac{\eta^2}{c^2}}} \Rightarrow \gamma^2 = \frac{1}{1 - \frac{1}{\gamma^2} \frac{\eta^2}{c^2}}$$

$$\Rightarrow \gamma^2 \left(1 - \frac{1}{\gamma^2} \frac{\eta^2}{c^2}\right) = 1 \Rightarrow \gamma^2 = 1 + \frac{\eta^2}{c^2}$$

$$\Rightarrow \gamma = \sqrt{1 + \frac{\eta^2}{c^2}}$$

II Momentum: Classically: mass \times velocity

So

$$\vec{p} = m \vec{v} \quad \text{or} \quad \vec{p} = m \vec{\eta} ?$$

The 1st def. would \Rightarrow conservation of momentum is not a possible law - if true in S, not true in S'.

Define: Relativistic momentum:

$$\vec{p} = m\vec{\eta} = \gamma m\vec{v} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

This means that \vec{p} is the spatial part of a 4-vector:
 $p^\mu = m\eta^\mu$

Note: E is not zero for a particle at rest
 $E = mc^2 \equiv R$ "Rest energy"

Then
 $T = E - R = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$
 "Kinetic energy."

$$= \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1) mc^2 \approx \frac{1}{2}mv^2$$

(non-relativistic)

Question: What's the zeroth P_{2/3} ("temporal") component?

$$p^0 = m\eta^0 = m\gamma c$$

Define: Relativistic energy

$$E = \gamma mc^2$$

or

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

SO,
 $p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$ "Energy-Momentum 4-vector"

Useful Fact:

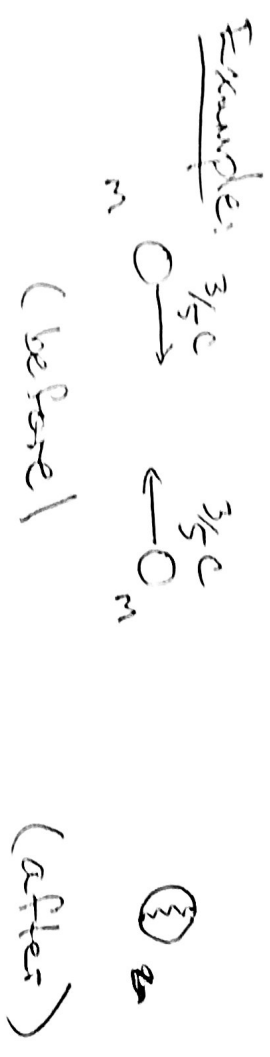
$$E^2 - p^2 c^2 = \frac{m^2 c^4}{1 - v^2/c^2} - \frac{m^2 v^2 c^2}{1 - v^2/c^2}$$

$$= \frac{m^2 c^4}{1 - v^2/c^2} \left(1 - \frac{v^2}{c^2}\right) = m^2 c^4$$

SO, $E^2 - p^2 c^2 = m^2 c^4$
 gets you directly from E^2 to p^2 .

III Conservation laws:

In any collision process (including decays) energy & momentum are conserved.



Question: What's the mass of the composite lump?

E.g. $\pi^0 \rightarrow e^+ + e^-$

Rest energy and mass decrease, while K.E (kinetic energy) increases.

def: In elastic collision is one in which K.E. is conserved (in practice, some particles out as in).

IV $m=0$

In classical mechanics, no such thing!
 $p = mv = 0$, $T = \frac{1}{2}mv^2 = 0$, $F = ma = 0$

(before) (after) $\frac{73}{3}$

$$\frac{2 m c^2}{\sqrt{1 - 9/25}} = M c^2$$

$$\Rightarrow M = \frac{2}{\sqrt{16/25}} m = \frac{10}{4} m = 2.5 m$$

Mass is not conserved, in general!

Note! All forms of 'internal' energy are included in the rest mass of a composite object ($m \frac{1}{2}c^2$)

Relativity: loophole! If $v=c$, then the denominators are also zero!
 $E = 0$? $p = 0$?

But, $E^2 - p^2 c^2 = m^2 c^4 = 0$

$$\Rightarrow E^2 = p^2 c^2$$

$$\Rightarrow E = \dot{p} c$$

In fact, they exist (photons, gluons, approximate for neutrinos)