Modern
Day 6

Today

I Last time

II Extended example of relativistic energy-momentum conservation & suggestions

I • Found useful technical result

$$\gamma = \sqrt{1 + \vec{\eta}^2/c^2}$$

• Defined relativistic momentum

$$\vec{p} = m\vec{\eta} = \gamma m \vec{v}$$

and energy

$$E = \gamma m c^2$$

We also collected these into the energy-momentum 4-vector

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right)$$

• Found that

$$E^2 - \vec{p}^2 c^2 = m^2 c^4$$

• With these def.s:

$\boxed{\text{In any collision process (including decays) energy \& momentum are conserved.}}$

Surprisingly, this conservation law is all that is needed to analyze many relativistic collision processes. While it can be hard to believe, no more information is needed.

Throughout the couse we will treat the neutrinos as massless unless otherwise stated.

greek mu.

μ^- : muon, like a heavy electron

ν_μ : muon neutrino, little neutral one
greek nu

$m_\mu = 105.7 \frac{MeV}{c^2} = 1.884 \times 10^{-28}$ kg

$m_{\nu_\mu} \sim \frac{1}{10}$ eV $= 1.78 \times 10^{-37}$ kg

9 orders of mag.

neglect and treat $m_{\nu_\mu} \approx 0$.

I **Example**: $\pi^- \to \mu^- + \bar{\nu}_\mu$ (anti particle)

$$\Rightarrow \vec{P}_\nu = -\vec{P}_\mu \Rightarrow |\vec{P}_\nu| = |\vec{P}_\mu|$$

So,
$$m_\pi c^2 = E_\mu + |\vec{P}_\mu| c$$

But,
$$P_\mu^2 c^2 = E_\mu^2 - m_\mu^2 c^4$$

$$\Rightarrow |\vec{P}_\mu| c = \sqrt{E_\mu^2 - m_\mu^2 c^4}$$

Then
$$m_\pi c^2 = E_\mu + \sqrt{E_\mu^2 - m_\mu^2 c^4}$$

$$\Rightarrow (m_\pi c^2 - E_\mu)^2 = E_\mu^2 - m_\mu^2 c^4$$

(before) (after)

What's the energy of the muon? Conservation of energy:

$$m_\pi c^2 = E_\mu + \underbrace{E_\nu}_{|\vec{P}_\nu| c}$$

Cons. of momentum

$$0 = \vec{P}_\mu + \vec{P}_\nu$$

$$\Rightarrow m_\pi^2 c^4 - 2 m_\pi c^2 E_\mu + \cancel{E_\mu^2} = \cancel{E_\mu^2} - m_\mu^2 c^4$$

$$\Rightarrow \boxed{E_\mu = \frac{(m_\pi^2 + m_\mu^2)}{2 m_\pi} c^2}$$

Its momentum?

$$|\vec{P}_\mu| = \frac{1}{c} \left(\frac{(m_\pi^2 + m_\mu^2)^2}{4 m_\pi^2} c^4 - m_\mu^2 c^4 \right)^{1/2}$$

$$= c \left(\frac{m_\pi^4 + 2 m_\pi^2 m_\mu^2 + m_\mu^4 - 4 m_\pi^2 m_\mu^2}{4 m_\pi^2} \right)^{1/2}$$

$$\Rightarrow |\vec{P}_\mu| = c\left(\frac{(m_\pi^2 - m_\mu^2)^2}{4m_\pi^2}\right)^{1/2}$$

$$= \boxed{\frac{m_\pi^2 - m_\mu^2}{2m_\pi}\, c}$$

Suggestion 1: To get the E of a particle given its $\vec{p}$ (or vice versa), use the invariant

$$E^2 - |\vec{p}|^2c^2 = m^2c^4$$

Suggestion 2: If you know E and $\vec{p}$ of a particle and you want $\vec{v}$, use

$$\vec{v} = \frac{\vec{p}c^2}{E},$$

Q: What is the muon's speed? Well,

$$E = \gamma mc^2, \quad \vec{p} = \gamma m\vec{v}$$

So,

$$\frac{\vec{p}}{E} = \frac{\vec{v}}{c^2}$$

Then,

$$v_\mu = \frac{|\vec{p}|_\mu c^2}{E_\mu} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}\, c$$