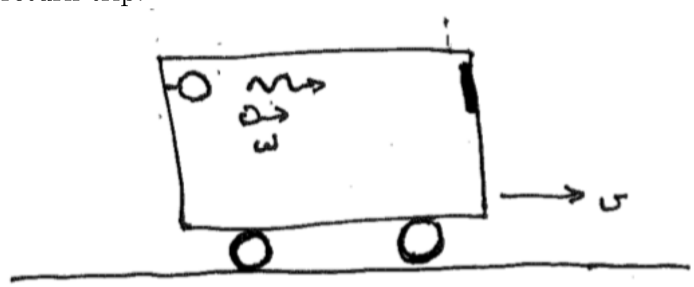
Homework 1

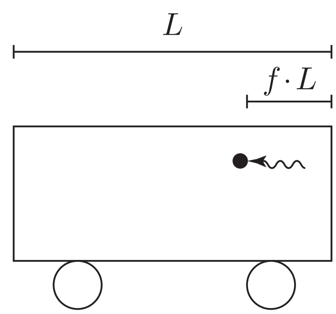
Due Thursday, September 10th at 10pm

Download and read Chapter 4 of Hartle's book.

Problem 1 (A rare train argument! Due to N. D. Mermin.) Suppose that there is a light bulb at the back of the train and a mirror at the front (like in our derivation of length contraction in class). This time imagine that we've sent a massive particle to race the light beam. Take the speed of this particle to be w with respect to the ground and assume that $w \geq v$, with v the speed of the train. (Why would it be uninteresting to consider $w < v$?) According to Einstein we also have $w < c$, consequently the light beam will reach the front of the train first, reflect and then encounter the massive particle on its return trip.



(a) The particle and light beam meet behind the front of the train. Consider this place on the train to be a fraction f of the total length of the train behind the front (see the sketch below). Will an observer on the train and an observer on the ground agree on the fraction f? If so, why and if not, why not?



(b) Calculate the fraction f according to an observer on the ground. [Hints: The algebra can get overwhelming here. Be sure to simplify your equations as much as possible at each step before proceeding to the next step. *Answer:* $f = \frac{(c-w)(c+v)}{(c+w)(c-v)}$.]

(c) Calculate the fraction f according to an observer on the train. You'll have to introduce one more speed for this part, call it u. What is the physical interpretation of u? There are at least two very different ways to do this part, try to find more than one. [Hint: The answer here is even simpler than in part (b).]

(d) In (a) you may have argued that f is frame independent, if so, what does setting the values of f from parts (b) and (c) equal to each other tell you about relativistic velocity addition?

Problem 2 Outlaws are escaping in their getaway car, which moves at $\frac{3}{4}c$, chased by the police, moving at only $\frac{1}{2}c$. Realizing they can't catch up, the police attempt to shoot out the tires of the getaway car. Their guns have a muzzle velocity (speed of the bullets relative to the gun) of $\frac{1}{3}c$.

(a) Does the bullet reach its target according to Galileo?

(b) Does the bullet reach its target according to Einstein?

Problem 3 Cosmic ray muons are produced high in the atmosphere (at 8000 m, say) and travel toward the earth at very nearly the speed of light ($0.998c$, say).

(a) Given the lifetime of the muon (2.2×10^{-6} sec), how far would it go before disintegrating, according to pre-relativistic physics? Would the muons make it to ground level?

(b) Now answer the same question using relativistic physics. (Because of time dilation, the muons last longer, so they travel farther.)

(c) Now analyze the same process from the perspective of the muon. (In its reference frame it only lasts 2.2×10^{-6} sec; how, then, does it make it to ground?)

(d) Pions are also produced in the upper atmosphere. [In fact, the sequence is proton (from outer space) hits proton (in atmosphere) $\rightarrow p + p +$ pions. The pions then decay into muons: $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$; $\pi^+ \rightarrow \mu^+ + \nu_\mu$.] But the lifetime of the pion is much shorter, a hundredth that of the muon. Should the pions reach ground level? (Assume that the pions also have a speed of $0.998c$.)

Problem 4 According to clocks on the ground, two streetlights A and B situated 4 km apart were turned on precisely at 8:00 pm EST:

(a) Which one turned on first according to passengers on a high-speed train moving from A straight toward B at a speed of $3/5c$?

(b) How much later (in seconds) did the other light turn on?

(c) In the frame of the earth, are the events corresponding to the lights turning on space-like, light-like, or time-like separated?

(d) How about in the frame of the train?

Problem 5 In class we have proved the time dilation formula

$$\Delta t' = \frac{1}{\gamma}\Delta t,$$

where the notation matches the one in our class notes. Let's practice your linear fitting skills in Python using this formula.

First generate a list of input time intervals, Δt's, in the frame S. Suppose that a high-speed train car moves through S with speed $v = 1/3c$. Use your input list of Δt's to generate a list of corresponding time intervals, $\Delta t'$'s in the train's moving frame, S'. Plot $\Delta t'$ vs. Δt and find a linear fit to the data. Before looking at the number, predict the slope of the best fit line. Does your prediction agree with the output of your linear fit code?

Here you can submit a screen shot of your best fit plot, your prediction for the slope, and the slope the computer produces.