Homework 4

Due Thursday, October 8th at 11:59pm

Read Hecht's Ch. 2. (I also encourage you to read Ch. 1, which is a very nice historical summary. However, this is not required.)

Problem 1 (a) A violin is submerged in a swiming pool at the wedding of two scuba divers. Given that the speed of compression waves in pure water is 1498 m/s, what is the wavelength of an A-note of 440Hz played on the instrument?

(b) Consider a pulse described in terms of its displacement at $t = 0$ by

$$\psi(x,t)\Big|_{t=0} = \frac{C}{2+x^2},$$

where C is a constant. Draw the wave profile. Write an expression for the wave, having a speed v in the negative x-direction, as a function of time t. If v =1 m/s, sketch the profile at $t = 2$s. (You are welcome to use python for the scketchs.)

Problem 2 Determine which of the following describe traveling waves (A, a, and b are constants):

(a) $\psi(y,t) = e^{-(a^2y^2+b^2t^2-2abty)}$

(b) $\psi(z,t) = A\sin(az^2 - bt^2)$

(c) $\psi(x,t) = A\sin\left[2\pi\left(\frac{x}{a}+\frac{t}{b}\right)^2\right]$

(d) $\psi(x,t) = A\cos^2 2\pi(t-x)$

For the cases that are traveling waves: (i) draw the profile (you can use Python to do this if you like), (ii) find the speed, and (iii) find the direction of motion.

Problem 3 Towards the end of your reading for this week there is a description of plane waves in 3D. Use this material as a reference to write an expression in Cartesian coordinates for a harmonic plane wave of amplitude A and frequency ω propagating in the direction of the vector $\vec{k}$, which in turn lies on a line drawn from the origin to the point $(4,2,1)$. [Hint: First determine $\vec{k}$ and then dot it with $\vec{r}$.]

Problem 4 For each of the following numbers, (a) $z = 1 - i\sqrt{3}$, (b) $w = 4\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$, and (c) $q = \sqrt{2}e^{-i\pi/4}$, first visualize where it is in the complex plane. With practice you can even find x, y, r, and θ in your head. Then plot the number and label it in the following five ways: (i) (x,y), (ii) $z = x+iy$, (iii) (r,θ), (iv) $z = r(\cos\theta + i\sin\theta)$, (v) $z = re^{i\theta}$. Also plot the complex conjugate of the number.

For (d) $z = \frac{1}{i-1}$ and (e) $w = \frac{3+i}{2+i}$ first simplify each number to the $x+iy$ form or to the $re^{i\theta}$ form. Then plot the number in the complex plane.

(f) Find the absolute value of $z/(z^*)$.

(g) Solve for all possible values of the real numbers x and y in the equation $\frac{x+iy+2+3i}{2x+2iy-3} = i+2$.

Problem 5 In class Monday, we will use the complex system of writing waves, $\psi(x,t) = A\cos(kx-\omega t+\phi) \rightarrow \tilde{\psi}(x,t) = Ae^{i(kx-\omega t+\phi)}$, to show that the intensity of a superposition of two harmonic waves with the same frequency and different phases is dependent on the cosine of their phase differences. Consider two waves with different frequency, wavenumbers and phase, but the same speed: $\psi_1(x,t) = A_1\cos(k_1x-\omega_1t)$ and $\psi_2(x,t) = A_2\cos(k_2x-\omega_2t+\phi)$, with $\omega_1/k_1 = \omega_2/k_2 = c$. The intensity of a wave (or a quantity proportional to its intensity) is $I(x,t) = \frac{1}{\tau}\int_0^\tau \psi(x,t)^2 dt = \frac{1}{\tau}\int_0^\tau [\frac{1}{2}(\tilde{\psi}(x,t)+\tilde{\psi}(x,t)^*)]^2 dt$, where $\tilde{\psi}(x,t)^*$ is the complex conjugate of $\tilde{\psi}(x,t)$ and the τ interval must be an integer number of cycles for harmonic waves, that is, a multiple of the period.

(a) Use the complex notation formalism to show that the intensity for $\psi_1(x,t)$ alone is proportional to A_1^2 and the intensity of $\psi_2(x,t)$ alone is proportional to A_2^2.

(b) Use the complex notation formalism to show that the intensity of $\psi_1(x,t)+\psi_2(x,t)$ is equal to the sum of the intensities of the individual waves (i.e. there is no interference). Here the time-average interval τ should be an integer number of cycles of both $\psi_1(x,t)$ and $\psi_2(x,t)$.

(c) Finally use the complex notation to calculate the intensity of the superposition of two harmonic waves with the same frequency and wavenumber, different phases and amplitudes, and going in opposite directions: $\psi_1(x,t) = A_1\cos(k_1x-\omega_1t) + A_2\cos(k_1x+\omega_1t+\phi)$.