Homework 5

Due Thursday, October 15th at 11:59pm

Read your Lyons' text, Secs. 2.1-3 pp 44-51.

Problem 1 On average, the total electromagnetic power radiated by the Sun, its so-called *luminosity* (L), is 3.9×10^{26} W. Determine the mean amplitude of the electric field due to all the radiant energy arriving at the top of Earth's atmosphere (1.5×10^{11} m from the Sun).

Problem 2 A plane, harmonic light wave has an electric field intensity given by

$$E_z = E_0 \cos\left(\pi 10^{15} \text{ Hz} \cdot t - \frac{\pi 10^{15} \text{ Hz } x}{0.65c}\right).$$

Find: (a) the frequency of the light, (b) its wavelength, and (c) the index of refraction of the glass.

Problem 3 (a) Is the distance that yellow light travels in water (where $n = 1.33$) in 1.00 s?

(b) A 500-nm lightwave in vacuum enters a glass plate of index 1.60 and propagates perpendicularly across it. How many waves span the glass if it's 1.00 cm thick?

(c) Yellow light from a sodium lamp ($\lambda_0 = 589$ nm) traverses a tank of glycerin (of index 1.47), which is 20.0 m long, in a time t_1. If it takes a time t_2 for the light to pass through the same tank when filled with carbon disulfide (of index 1.63), determine the value of $t_2 - t_2$.

Problem 4 Examine the table and graphs of Figures 6 and 7 at the webpage:

philiplaven.com/p20.html,

which shows details of how electromagnetic waves travel in water. Note that the x-axes are in terms of the wavelength of these waves in vacuum. Answer the following questions for electromagnetic waves that have wavelengths of (i) 400nm, (ii) 700nm, (iii) 1mm, and (iv) 10 m in vacuum:

(a) What is the speed of each of these waves in water?

(b) What is the (real) wavenumber of each of these waves in water?

(c) What is the angular frequency of each of these waves in water.

(d) Suppose each of these waves have intensity I_o when entering the water. How far does each travel in water before the intensity is reduced to $10^{-10} I_o$? (This reduction would render an almost overwhelmingly bright light imperceptible to the human eye). Make a connection between your answers and why water appears blue.

Please explain how you answer each type of question in words, but collect all your answers in a single table. This will be helpful for you to see comparisons between your results.

Problem 5 Rainbows are possible because water has an index of refraction that depends on wavelength, hence, when light enters water different wavelengths bend different amounts. In 1871

Sellmeier derived an approximate model for the wavelength dependent index of refraction of a transparent medium:

$$n^2 = 1 + \frac{A_1 \lambda^2}{\lambda^2 - \lambda_0^2},$$

where A_1 is a numerical constant and λ_0 is a 'natural frequency' for the electrons in the material, such that $\lambda_0 f_0 = c$. This was an improvement on a previous approximate formula invented by Cauchy, who modeled $n(\lambda)$ by

$$n = C_1 + C_2/\lambda^2 + C_3/\lambda^4 + \cdots .$$

Use Sellmeier's formula to derive Cauchy's formula. [Hints: You will want to use Taylor expansion to do this. Identify a small parameter ϵ and Taylor expand the second term in Sellmeier's formula around $\epsilon = 0$. Next take the square root to find n and Taylor expand again.]