Homework 6

Due Thursday, October 22nd at 11:59pm

Read Sections 1.1-1.3 of Chatper 1 of Schroeder's book *Thermal Physics*.

Problem 1 The relation between angular frequency ω, wavenumber k, and amplitude A for waves in deep water is often modeled by the relation $\omega^2 = gk[1 + (kA)^2]$, where g is the acceleration due to gravity. Using this model,

(a) What is the phase velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(b) What is the group velocity in m/s of a group of waves with an amplitude of 1m and wavelength of 2m?

(c) Is it possible to have a group of waves with the same phase and group velocities? If so, find a k and A for which this is true.

Problem 2 This problem has the same setup as **Problem 4** of the last homework.

(a) Use the data of Fig. 4 on the webpage `philiplaven.com/p20.html` to figure out a sketch of $\omega(k)$ vs k for these wavelengths. [Hint: Neither of the axes is what you need yet, so this will take several steps.] Your sketch should have enough detail of the increase/decrease and curvature of $\omega(k)$ to see whether the group velocity is increasing or decreasing with k.

(b) Use the table of Fig. 6 to find the phase velocity of a 400nm (in vacuum) harmonic wave and a 425nm (in vacuum) harmonic wave.

(c) Estimate the group velocity of the superposition of these two harmonic waves. Is it faster or slower than their phase velocities?

(d) Find the phase velocity of a 675 nm (in vacuum) harmonic wave and a 700 nm (in vacuum) harmonic wave.

(e) Estimate the group velocity of the superposition of the two harmonic waves in (d). Is it faster or slower than their phase velocities?

(f) Which is faster, the group velocity of the first pair of harmonic waves or the second pair? Explain how one can see this from the sketch you drew in (a).

Problem 3 Find the first 5 terms of the Taylor expansions about $x = 0$ for each of the following functions:

(a) $\cos(kx)$ [k is a constant]

(b) $\sin(kx)$ [k is a constant]

(c) $e^{-\alpha x}$ [α is a constant]

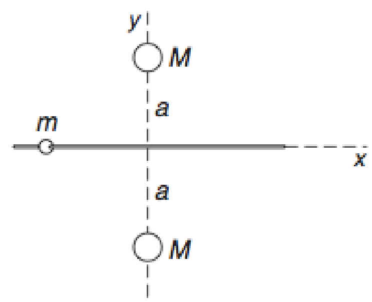
(d) e^{ix} [i is the imaginary unit $\sqrt{-1}$]

(e) Figure out how to write the general term in the expansions from (a), (b), and (d).

(f) Use your answer to (e) to prove the Euler relation $e^{ix} = \cos x + i \sin x$.

The next two problems draw on your background in mechanics. In each case different things that you know about gravity. In both cases you want to understand how the systems compare to a harmonic oscillator. In the first problem, use the ideas that you have been learning about Taylor series expansion of potential energy. In the second, compare a force analysis to the force on a harmonic oscillator.

Problem 4 A bead of mass m slides without friction on a smooth rod along the x-axis. The rod is equidistant between two spheres of mass M. The spheres are located at $x = 0$, $y = \pm a$ as shown, and attract the bead gravitationally. Find the frequency of small oscillations of the bead about the origin.



Problem 5 A glass tube bent in the shape of a U contains water of density ρ. Initially the water is in equilibrium and fills each arm of the tube to a height h. Take the total length of the column of water to be ℓ. Then the whole column of water is displaced so that now the left arm contains water up to a height y above the equilibrium height and the water begins to oscillate. Assume that the tube has a circular cross section of radius r. What is the period of oscillations for this water column?